How to mimic DB-like benefits in a DC product

Master thesis Economics and Finance of Aging

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Abstract

A classic defined benefit scheme delivers a real annuity at retirement, which is considered to be the optimal pension provision for individuals. The unsustainability of these schemes combined with new regulation is driving a worldwide shift towards defined contribution solutions, which do not deliver a guarantee and thus a suboptimal pension. This thesis shows through a simulation study that using financial derivatives there is a way to combine the best of the two worlds into an individual pension product that has clearly defined ownership while delivering a DB like pension.

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1. Introduction

People seek to maximize their well-being, not at a single point in time, but over time. (Barr and Diamond, 2008). Following Yaari (1965), Milevsky (2004) states that 'in a life cycle model without bequest motives, retirees should convert their liquid assets into life annuities that provide longevity insurance and protection against outliving one's money.' This should maximize lifetime utility following the life cycle hypothesis originally developed by Ando and Modigliani (1963)[1]. A nominal annuity does not achieve the optimal level of welfare because the purchasing power of the benefits might decrease over time due to inflation. Brown and Nijman (2011) conclude that individual welfare is received from consuming real goods, and that consumption smoothing over the life cycle is meant in real terms. A real annuity at retirement day is then the optimal solution.

For a long time, defined benefit pension schemes have provided real annuity benefits which maximized welfare for the retirees. These schemes were offered by companies to their employees, with the employer promising to take on the risk. However, this is only worth something when the company is able to hold on to that promise which is often questionable during economic downturn when the guarantee is needed most. And then there is the probability of default of the company, leaving the pension plan members behind without any guarantee[2]. Under current IFRS the pension liability has to be made explicit on the balance sheet of the company, exposing the true risk underlying the DB scheme. Many companies are either unable or unwilling to bear the risk of the guarantees, which makes DB plans unsustainable in the long run. This is one of the main reasons for a worldwide shift towards defined contribution systems (Baculik, 2010). Other reasons, such as lack of portability to other employers are noted for driving the demise of DB by Broadbent et al. (2006). The upcoming IORP[3] and Solvency II regulatory frameworks are likely to contain tougher regulation regarding guarantees in pension arrangements, accelerating the process of moving away from DB and guaranteed insurance solutions.

The move from DB to DC has a negative consequence for plan members, because DC schemes are nothing more than a savings account and at first sight do not solve the pension problem of individuals. Unlike the integrated product offered by a DB scheme, there is a complete disconnect between the accumulation and decumulation phase in a DC pension plan. The latter is by design solely aimed at accumulation without any regard to the decumulation phase. Bernstein (2008) notes that it appears 'as if each link of the DC system is designed in isolation from the whole chain.' Often the retiree has to take the lump sum that he gets at retirement to an annuity provider. At this critical moment, there is a huge conversion risk because the retiree is first forced to sell one's assets, which might be at a low value. Next, the forced purchase of an annuity at that date is subject to interest rate risk because low interest rates will result in low annuity rates leading to a life-time lower pension income (Blake, 2008). This turns picking the retirement date into a lottery ticket with good and bad outcomes, which is not desirable for retirement benefits needed to maintain a standard of living. Defined contribution does have some characteristics which speak in its favor, as noted by Benartzi and Thaler (2007). The portability problem is solved because the individual account is not

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[1] This is under the assumption of no bequest motives

[2] In some countries there is a government rescue fund for these events.

[3] Institutions for occupational retirement provision, an EU directive for pan-European pension funds.

subject to unclear ownership structures which are common in DB schemes, providing greater transparency. Clear ownership prevents intergenerational transfers which are out of control of the individual in systems where trustees can change the pension plan, such as in the Netherlands. DC offers freedom of choice in investment strategy, even though this comes with the responsibility to choose wisely which is hard for most people, even for Nobel laureates. Harry Markowitz, the 'father' of modern portfolio theory, is quoted to divide his contributions fifty-fifty over stocks and bonds even though he admitted that 'he should have calculated the historic covariances of the assets classes and draw an efficient frontier' (Zweig, 1998). Benartzi and Thaler (2001) report that a majority of the participants chooses this asset allocation, while Lusardi and Mitchell (2011) find widespread financial illiteracy among older Americans. Libby, Munnel, Prinzivalli and Soto (2006) show that half of the 401(k) participants do not diversify between asset classes. For these reasons it is questionable whether putting the burden of deciding over the asset allocation to the participant is optimal, especially since the risk of falling short on the pension promise rests completely with the participant (Blake et al. 2001). This includes the conversion risk noted above since, contrary to a DB pension, a DC plan does not involve a contractual obligation about the size of the pension and therefore is not a risk to the provider. In this matter, there is an information asymmetry between suppliers and consumers, which can be and is exploited. The 2008 RSA report noted that fees are deliberately shown on an annual basis to hide the compounding effect which is a big part of total costs. The report argues for a mandatory percentage-of-final-value cost measure in order to protect the consumer. Another danger to consumers is the enormous amount of choice in funds to invest in, which has the surprising consequence of low participation rates. Schwartz (2005) reports that participation goes down two percent for every ten funds offered more in a 401(k) retirement plan due to decision paralysis, which can be considered a threat to optimal pension decision making by individuals. All these different funds charge separate fees which explains part of the high total costs. These costs are often not transparent and are used for active management of the investments while there is no evidence to justify this kind of asset management (Blake, 2008).

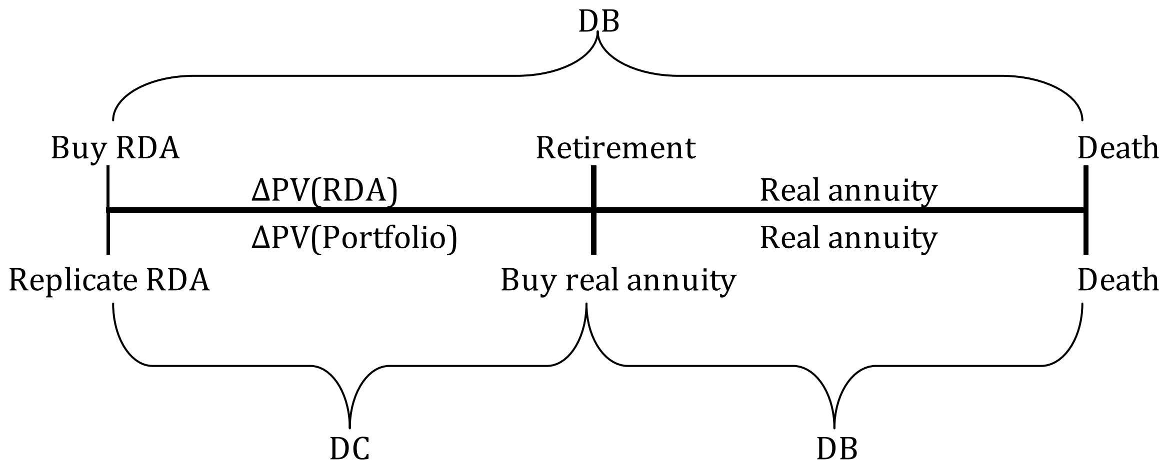
We can thus conclude that traditional DC plans deliver a product which is far from optimizing welfare, while putting large risks on the participants in a setting the average consumer has problems understanding, at a price which is often said to be too high. Employers have proven to be incapable of bearing the risk of a DB scheme, which makes this solution unsustainable. The goal of this thesis is then to approach the optimal pension product as close as possible under the constraints of the current financial order. Concretely, this means replicating a real deferred annuity (RDA) since that is the product that is bought in a classic DB scheme[4], and is in the literature considered as the best contract from a welfare perspective. Given the current trend, this product has to be achieved within the DC framework regarding the accumulation phase.

The research question of this paper is: Can we create an investment strategy that approximately replicates a real deferred annuity? If the answer is yes then we have an attractive solution that can be implemented in a DC framework for those who do not have access to a DB plan.

Figure 1 on the next page compares classic DB with the proposed solution. The top side is the classic DB scheme in which a RDA can be bought, while the bottom side depicts how the same retirement product should be achieved in the DC framework.

[4] We abstract from DB schemes with conditional indexation

Figure 1: Classic DB vs DC space RDA



The top side of the figure denotes a classic DB scheme, whereas the bottom side retains the goal of DB benefits but with a DC accumulation phase. The ΔPV refers to the change in present value. If $\Delta PV_t(Portfolio) = \Delta PV_t(RDA)$ at every t during the accumulation phase, the DC investment portfolio will at any time equal the theoretical value of a real deferred annuity which is implicitly bought in a DB scheme. One can then use the DC investment portfolio to buy a real annuity at retirement day.

Given that the goal of retirement saving is the ability to buy a real annuity at a certain date in the future, the goal of any DC solution should be to equal the upper left side of the time line so that:

$$\Delta PV_t(Portfolio) = \Delta PV_t(RDA) \quad for\ every\ t\ (time) up\ to\ retirement$$

$$PV = present\ value\ and\ \Delta\ is\ the\ change\ from\ period\ t\ to\ period\ t+1$$

If this equation holds DC equals DB on an individual account. Similar to the DB scheme, this DC solution does not suffer from the conversion risk that is such a major backdrop of classic DC schemes. The solution also eliminates the difficult choices for individuals common in DC such as the asset allocation. Zvi Bodie is quoted on this topic as: ‘Finance is like medicine: people do not like it, rely on expert advice and see choice as a mixed blessing.’ (Bodie and Prast, 2007). On top of this, Merton (2003) phrases it as: ‘New technology and deregulation have left households with the responsibility for making important and technically complex micro financial decisions involving risks that they had not had to make in the past, are not trained to make in the present, and are unlikely to execute efficiently in the future, even with attempts at education.’ Individual choice is putting the burden of responsibility on the least qualified, away from where the necessary expertise is located. This can be prevented if there is a strong and proper default with limited choice for the minority that wants a different asset allocation.

Together with the advantages of DC this strategy may be a good individual replacement of the DB product. To achieve that, the RDA has to be replicated in the financial market. Replicating a RDA is completely dependent on three underlying variables, namely the nominal interest rate, the inflation rate, and mortality rates. If we are to replicate the value of the RDA at any time, these variables have to be hedged. The first two are hedged using financial derivative instruments, while mortality risk is left open in this individual setting. Section 2 provides more information on these variables and on the subject of costs. Section 3 describes the simulation methodology which tests different hedging strategies, after which Section 4 presents the results. Section 5 concludes.

2. Background

This section will provide more information on the three variables underlying the value of a real deferred annuity and the hedging methods. The involved risks and costs are also elaborated on.

Hedging nominal interest risk will be performed using the interest rate swap market, which is very liquid in the Euro zone and offers a range of maturities on swaps. The underlying Euribor curve is used as nominal discount curve for valuing the liabilities, as interest rate swaps are the most liquid instruments available. Usage of the Euro swap curve is common market practice, for example FTK[5] regulation in the Netherlands prescribes the use of this curve. The techniques used for building the discount curve are demonstrated in the appendix.

The second variable that has to be hedged is inflation, which can have a devastating effect on the purchasing power of nominal benefits if not hedged. Spierdijk and Umar (2011) list a bulk of literature on the inflation hedging capacity of different asset classes. The main hypothesis is that stocks are claims on real assets and should therefore be a good inflation hedge. However, most literature, including that of the above authors, finds a negative correlation between stocks and inflation or only a positive hedging capacity in the very long run. Campbell and Viceira (2005) report that inflation creates stock market mispricing that can have large effects at intermediate horizons, but in the very long run it corrects itself and stocks are able to hedge inflation risk. T-bills are often cited to be a good inflation hedge because the short maturity allows adapting to changing inflation levels, leaving only short term inflation risk. Campbell and Viceira (2005) remark that this same advantage does bring in reinvestment risk over longer horizons though. Brown et al. (2001) state that stocks and bonds have not been a good inflation hedge over the past seventy years and that there is no conclusive evidence about significant correlation with inflation. The idea of hedging inflation by looking for correlations with certain asset classes is a prime example of the asset management mindset. This might work well in expectation, but the tail and conversion risk is not managed which is one of the main problems in full-DC systems. Rodriguez (2011) already concluded that traditional asset management strategies fail to provide the product needed at retirement. He also showed that life cycle strategies, even though being the cornerstone of many DC plans according to Blake et al. (2001), do not solve the problem. The extensive range of literature on forecasting inflation and other variables is neither applicable here because an RDA replicating portfolio is not designed to exploit a view on the market, but to track the theoretical RDA value regardless of market conditions.

Instead we approach the research question using an insurance mindset and identify the break even inflation (BEI) rate as inflation measure, which is based on the harmonized index of consumer prices excluding tobacco (HICPxT) and is published by Eurostat (Kerkhof, 2005). This is a Euro zone aggregate which is not necessarily the same as a particular national inflation index usually named the consumer price index (CPI). HICP is a weighted average of the national CPIs which differ per country. The difference between the national CPI and the (Euro) HICP is generally referred to as mismatch risk. This term has more definitions, however: the difference between the actual consumption pattern and the CPI basket can also be referred to as mismatch risk. Huang and Milevsky (2011)

[5] FTK is the Dutch regulatory framework on pensions, based on IFRS in the sense that is prescribes market valuation of the liabilities.

report a structural difference between the average consumption pattern measured by the CPI and the experienced inflation by retirees. In the US the CPI-E measures the inflation of the consumption basket of the elderly, and is supposed to be a better approximation than the regular CPI measure which tends to be lower.[6]

Currently, the market for HICPxT linked products is the closest we can get because personalized inflation rates do not exist and market liquidity in other proxies is simply too low or not present. We cannot deny that there will be mismatch risk, but the main goal is protection against inflation shocks. If inflation was perfectly predictable, a nominal annuity with an escalation rate would suffice in hedging it. The assumption that inflation shocks will be reasonably correlated throughout a single currency area does not seem unrealistic, especially not given the long investment horizon. De Jong, Mens, Oerlemans and Potters (2008) show using Ecowin[7] data that the cumulative HICP and Dutch CPI from 1990 to 2008 are practically equal separate from a couple of temporary mismatches, so we can conclude that hedging on HICP is defendable. Regardless, the mismatch is much smaller than when using equities as inflation hedge. The instruments used to hedge against inflation shocks are zero coupon inflation swaps and are discussed in detail in the appendix, as is the construction of an inflation curve.

The third variable to which the value of the RDA is subjected is mortality. For an individual, longevity risk is a reason to annuitize wealth in order to protect oneself against outliving ones assets. An insurance company with a sufficiently big pool of annuity clients diversifies away the idiosyncratic longevity risk, but the mortality table can change over time leaving the systematic mortality risk. Next to this, annuity providers charge a selection premium, defined by Blake (2008) as ‘to cover the additional longevity risk of someone who purchases an annuity in comparison with a typical member of the population with the same sex and age.’ This is the result that only people who expect to live long will buy an annuity, known as adverse selection. For both these risks, a markup is charged as a risk premium, causing annuities to be unfairly priced if one uses the aggregate mortality table for calculating the present value. James and Vittas (1999) show that the money’s worth is much lower for real annuities than for nominal ones, concluding that ‘the biggest weakness of annuity markets seems to be its inability to deal with inflation protection in a low cost way.’ Brown et al. (2001) suggest that the higher cost for real annuities over nominal ones can be explained by a high degree of adverse selection, because real annuities are more backloaded[8] than nominal ones causing only people with very high subjective longevity to buy them. The selection premium charged by the insurance company is then higher than in case of a nominal annuity, to compensate for this effect. As stated earlier, Solvency II is going to require higher buffers for this kind of risk, further driving up the costs. Regardless of the cost, the availability of RDAs is far from what is needed for individuals to properly hedge longevity risk, interest rate risk and inflation risk. Blake (2008) accuses the insurance industry of having been ‘particularly unimaginative’ in designing products that solve this problem.

[6] One of the main reasons for the CPI-E to be higher concerns medical expenses. The aggregate cost of health care increases also due to technological progress that delivers new but more expensive treatments. Whether this can be referred to as normal price inflation is questionable, but it does impact the elderly in a similar fashion.
[7] Ecowin is a part of Reuters that provides economic and financial market data
[8] This means that a relatively bigger part of the cash flows occurs at the end of the product’s life.

If a 30 year real deferred annuity were available today, the markups for mortality risk would likely be much higher due to the longer period of uncertainty. Blake and Burrows (2001) confirm this by stating that 'deferred annuities are even more susceptible to mortality risk than immediate annuities'. In the proposed DC solution we therefore do not hedge mortality risk so that we can put the premium in the investment portfolio as a buffer against mortality shocks. Regardless of these cost, it would be difficult to trade a mortality hedge, even in the OTC market. Also, the underlying variable would be the mortality table of the whole population contrary to the annuity buyers which would introduce mismatch risk similar to inflation hedging, but then in another fashion. De Crom, de Kreuk, van Dijk, Vellekoop, and Vermeijden (2011) do present some structures that could be used to hedge mortality risk, but the amount of trades up to today can be counted on a few hands making secondary market liquidity non-existent.

Due to the long duration of pension products, yearly costs can have a major impact on the performance of the portfolio. Blair (2008) takes British unit trusts as an example to illustrate that the cost of one percent in yearly fees amounts up to 37 percent of the final value of the portfolio after 50 years. This illustrates the compounding effect that harms consumers discussed earlier. It is therefore imperative that operating and transaction costs are minimized because that is the simplest way to maximize return. A hedging strategy that is more complex is usually also more expensive because the spreads of complex products are larger and the execution of trades requires a more specialized treasury desk. A side effect is that the probability of operational errors is larger for complex products. Both these measures will be taken into account when assessing the performance of different hedging strategies, but they are not explicitly taken into account in the calculations. Non-linear products such as swaptions are left outside the considered alternatives, as the nature of option products (e.g. having an upside which comes at a price) does not fit with the objective of replicating an annuity under any circumstance. Also the involved costs and complexity are deemed beyond reach of this analysis.

As mentioned earlier, DB is an integrated product that delivers a real annuity. If DC is capable of delivering a portfolio which is worth the same as the DB delivered real annuity, one has the option instead of the obligation to annuitize the portfolio. This gives some flexibility in the usage of the funds, which is likely to be under legal and tax constraints however. Whether it is advisable from a behavioral view to allow people to take out part of the money as a lump sum is beyond the scope of this thesis. The advantages of an individual system are clear however: the ownership rights are clearly defined which is not the case in a collective DB scheme, so that (international) mobility is safeguarded. Currently, pension reform debates are widespread and also going on in the Netherlands. The social partners are discussing how to divide the pension assets, but not all groups are or feel represented in the discussion. From that sense, an individual suffers 'trustee risk' in a collective system, which is not present when owning an individual account. The best example of this concerns 'sleepers', people who built up pension rights in the past and now are employed somewhere else. This group is usually not explicitly represented in the board of trustees and have thus no say in the pension plan they are part of. The current DC systems are for sure not the best solution, but by improving upon it we might be able to deliver a second-best to DB.

Rodriguez (2011) already concluded that traditional asset management strategies fail to provide the product needed at retirement. His benchmark is the theoretical value of a RDA from purchase to retirement. This thesis builds on his work by using the RDA value benchmark in determining performance of different investment strategies. The aim here is approaching the benchmark by using a back to front approach in pension system design as advocated by multiple authors, including Blake (2008). This should translate into a portfolio that tracks the theoretical value of a RDA, regardless of market conditions. The prime extensions lie in the use of plain vanilla over-the-counter (OTC) derivatives and inflation-linked products in the investment portfolio. A limitation is that stochastic mortality tables are not part of the analysis, contrary to the work of Rodriguez (2011). Regardless, the risk-based portfolios should have much less tracking error than the asset management strategies commonly used today. As an example, Blake (2008) mentions protected annuity funds which deposit the largest part of the premium and use the remainder to buy call options on bond futures contracts which are supposed to protect against falling interest rates. This strategy is however aimed at buying a nominal annuity at retirement, but it does give us a starting point in attempting to replicate the RDA. We will start each subject with the nominal framework and then extend with inflation to get to the real framework. If this framework is successfully developed, it can be a starting point in determining the actual required premium of a DB pension. Working this out goes beyond scope here, the focus will be on developing the concept of the risk free asset. Section 3 starts with explaining the simulation methodology.

3. Simulation study

In order to test the performance of the different hedging strategies, the tracking error of the portfolio compared to the deferred annuity value is measured every year, with the last year obviously being the most important one as that is the point where the portfolio is used to buy retirement benefits. This tracking error is measured over 5000 scenarios so that the mean and standard error can be established accurately. In order to be clear on the methods used the valuation of annuities is discussed first. The scenario generation and testing methodology is also elaborated on.

3.1. Valuation of annuities

A nominal annuity is nothing more than a constant stream of cash flows for every period the buyer is alive, whereas a deferred annuity indicates that the first cash flow realizes at some time in the future. A real annuity is not a constant stream because the cash flows are indexed for inflation, making a real annuity more backloaded than a nominal one. If a real annuity is deferred, the cash flows should be inflated with the inflation over the full period from start to payout. Rodriguez (2011) only applies indexation starting from the first payout date, but that exposes the buyer to inflation risk during the deferral period which can be as long as 30 years. This risk is so substantial that indexation starting at payout could be futile, and therefore we include the inflation during the deferral period in the cash flow indexation. This is the only way to truly protect the buyer from inflation risk and is a necessary condition for a product claiming to be a replicate of DB benefits, which are usually indexed during the deferral period.[9]

In the background section it was already stated that the idiosyncratic longevity risk disappears in a sufficiently large pool of clients, so that there is no premium paid for this risk when buying an annuity.[10] The risk of the mortality tables changing is ignored, because we are unable to hedge it as discussed earlier. From a product perspective one could argue that the (high) mortality risk premium that should otherwise have been paid is put in the investment portfolio as a buffer against mortality shocks. The focus here is to hedge interest rate and inflation risk, so that individual mortality risk is not taken into account. Therefore we use a very specific mortality table which results in the client living 20 years into retirement and dying at 90 with certainty.[11] This does not cause any loss in generality because any other stream of cash flows can be hedged using the same method, but it simplifies operations considerably. Ideally, a longer period would have been used but the lack of availability of very long term interest rate instruments limits the total period to 50 years. This is because extrapolating the curve imposes problems in the hedging and valuation processes, delivering odd results. This will further be discussed in the sensitivity analysis. Another (harmless) simplification is that the notation and simulations are based on an end-of-year annual payout, while in real life this would usually be monthly. This is mostly done to lower the time it takes to run simulations, but it also makes notation easier.

To build intuition for the valuation we will start with valuing a nominal annuity, after which the RDA follows.

[9] The indexation in a DB scheme can also be based on wage inflation instead of price inflation, but that is ignored here as there are no derivatives based on wage indexation available.
[10] The adverse selection premium which is usually charged is not meant here.
[11] We take a futuristic view in which the retirement age is 70

3.1.1. Nominal deferred annuity (NDA) value

The value of a nominal annuity is the present value of all the future cash flows. The present value of a cash flow is obtained by multiplying it with its respective discount factor. The latter is denoted as:

$$D_{t,k}^{N} = Euribor\ discount\ factor\ from\ time\ t\ to\ time\ k\ with\ N\ being\ 'Nominal'$$

Euribor refers to the swap curve from which the discount factors are extracted. This is done using the curve methodology described in the appendix. One could argue that discount factors should be derived from the market prices of zero coupon bonds, but these are not available for very long maturities and are less liquid than swaps. The present value of a nominal annuity at time t is then valued as:

$$PV_t(NA) = B * \sum_{k=1}^{M} D_{t,k}^{N}\ where\ B\ denotes\ nominal\ benefits\ and\ M\ is\ mortality\ date \tag{1}$$

The cash flow stream can of course be deferred in order to obtain the nominal deferred annuity:

$$PV_t(NDA) = B * \sum_{k=R}^{M} D_{t,k}^{N}\ where\ R\ denotes\ retirement \tag{2}$$

The start of the annuity is usually at retirement, so this will be used as the starting point as we are basically valuing pension benefits. In the simulation, retirement will be fixed at 70 and the mortality date at 90, leaving a 20 year stream of cash flows. Since a good pension is indexed for inflation, we will continue to the real pension.

3.1.2. RDA valuation

For valuing a real cash flow a real term structure is required. The Fisher hypothesis states that the real interest rate is the nominal rate minus the expected inflation (Fisher, 1930). We will not use real interest rates but explicitly distinguish between nominal interest and inflation rates because it is intuitively helpful and because real interest rate swaps are not traded in the market. Recall from section 2 that the break even inflation (BEI) is used as inflation measure. It should be noted that the break even inflation is not necessarily equal to the expected inflation though. There is a risk premium embedded in the BEI rate, which compensates the receiver counterparty for taking on the risk of a higher inflation (Kerkhof, 2005). Nevertheless, we use BEI to inflate the future nominal payments which consequently are likely to be too high. However, there is no consensus on the size of the risk premium and BEI is the only inflation measure with enough potential liquidity for hedging purposes. The difference is therefore not taken explicitly into account, keeping in mind that providing too much indexation will certainly not harm the client as it comes closer to wage indexation which is applied in some DB schemes.

Swaps on the break even inflation (BEI) are frequently traded and combined with nominal interest rate swaps a real swap can be replicated. This approach will be used to hedge the value of the RDA. As with the interest rates, the inflation curve building process is explained in the appendix.

Using both the zero and BEI curve, real annuities that start today can be valued as[12]:

$$PV_t(RA) = PPA_t * \sum_{k=1}^{M} \frac{D_{t,k}^{N}}{D_{t,k}^{BEI}} \quad where\ N = nominal \tag{3}$$

$$PPA_t = \ Purchasing\ Power\ Amount\ at\ time\ t$$

Note that the discount factors have a superscript denoting the curve from which they are derived. The PPA is the real amount that should be paid out at every payment date, in today's nominal terms. The nominal payout for a period is calculated by dividing the PPA by the inflation discount factor of today to the payment date. The resulting number is then discounted to today by multiplying it by the nominal discount factor for the respective period. The value of the RA is then the sum of all discounted inflated payments.

Including the deferral period leads to the RDA present value:

$$PV_t(RDA) = PPA_t * \sum_{k=1}^{M} \frac{D_{t,R+k}^{N}}{D_{t,R+k}^{BEI}} \tag{4}$$

$$Where\ R = Retirement\ and\ t = current\ time$$

The calculation of the RDA is similar to that of the RA but now the benefits are inflated and discounted taking the deferral period into account. This valuation formula will be used when calculating the deltas with respect to the two underlying curves. In the simulation the deferral period will be 30 years, with 20 years of benefits afterwards. This makes $R = t + 30\ and\ M = t + 50$ at the start of the replication. The next subsection describes how the replication is performed.

3.2. Delta hedging

The value of the RDA in this framework is determined fully by the interest rate and inflation term structure. In order to allow the agent to buy a real annuity at retirement, his investment portfolio should react similarly to these two determinants as the RDA does, as explained in figure 1 in the introduction. To be able to hedge the RDA, its exposure to the two variables should be quantified, which is done by determining the deltas of the product. These deltas can then be used to determine the notional amounts of the hedging instruments so that a hedge can be set up.

The deltas (Δ) of a RDA with regard to its two underlying curves can be determined by taking the first derivative with respect to the underlying variable and is denoted as:

$$RDA\ par\ swap\ rate^{13}\ exposure\ to\ interest\ rate\ swap\ rate\ i = \ \Delta_i^N = \frac{\partial RDA}{\partial K_i}$$

$$RDA\ break\ even\ inflation\ exposure\ to\ inflation\ swap\ rate\ i = \ \Delta_i^{BEI} = \frac{\partial RDA}{\partial BEI_i}$$

Where i denotes one of the 18 instruments from one to 50 years used to build the underlying curve. The 18 instruments are listed in the appendix and are visible in table 1 below.

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[12] For RAs that started in the past, the accrued inflation from the start date to today has to be taken into account to calculate the benefits.

[13] The par swap rate is denoted by K to prevent confusion with zero rates

The delta exposure of the RDA to its underlying determinants is now known, so that a delta hedge can be set up. Summing up all the RDA par swap deltas would give us the duration of the liabilities. Based on the duration we could set up a hedge using a single swap of for example 30 years maturity. This approach imposes the assumption that the interest rate curve moves parallel when a shock occurs. As changes in the shape of the curve are frequently observed, this is not a plausible assumption for both the interest rate and the BEI curve. Best hedge effectiveness can then be achieved by hedging all deltas separately with their respective instrument, but this brings complexity and inefficiency. We are thus on a tradeoff between hedge effectiveness versus complexity and costs.

The deltas are divided into buckets, and then each bucket is hedged separately with the bucket instrument. The distribution of the deltas over the buckets and the hedging instruments are shown in table 1.

Table 1: Bucket distribution (7 buckets)

Bucket	RDA Nominal deltas	Bucket Instrument	RDA BEI deltas	Bucket Instrument
1	$\Delta_1^N + \Delta_2^N$	K_{2y}	$\Delta_1^{BEI} + \Delta_2^{BEI}$	BEI_{2y}
2	$\Delta_3^N + \Delta_4^N + \Delta_5^N$	K_{5y}	$\Delta_3^{BEI} + \Delta_4^{BEI} + \Delta_5^{BEI}$	BEI_{5y}
3	$\Delta_6^N + \Delta_7^N + \Delta_8^N + \Delta_9^N + \Delta_{10}^N$	K_{10y}	$\Delta_6^{BEI} + \Delta_7^{BEI} + \Delta_8^{BEI} + \Delta_9^{BEI} + \Delta_{10}^{BEI}$	BEI_{10y}
4	$\Delta_{15}^N + \Delta_{20}^N$	K_{20y}	$\Delta_{15}^{BEI} + \Delta_{20}^{BEI}$	BEI_{20y}
5	$\Delta_{25}^N + \Delta_{30}^N$	K_{30y}	$\Delta_{25}^{BEI} + \Delta_{30}^{BEI}$	BEI_{30y}
6	$\Delta_{35}^N + \Delta_{40}^N$	K_{40y}	$\Delta_{35}^{BEI} + \Delta_{40}^{BEI}$	BEI_{40y}
7	$\Delta_{45}^N + \Delta_{50}^N$	K_{50y}	$\Delta_{45}^{BEI} + \Delta_{50}^{BEI}$	BEI_{50y}

The distribution when fewer buckets are used is shown in appendix 7.4; the effects on the hedge effectiveness are discussed in the results.

In both the inflation and interest rate markets, a par swap only has delta exposure to its maturity swap rate. When calculating the delta of a par swap with a notional of one, we obtain the unit delta of the swap:

$$unit\ delta\ of\ interest\ rate\ swap\ (IRS)\ i = U_i^N = \frac{\partial IRS_i}{\partial K_i}\ with\ i\ being\ the\ tenor$$

$$unit\ delta\ of\ inflation\ linked\ zero\ coupon\ (ILZC)\ swap\ i = U_i^{BEI} = \frac{\partial ILZCS_i}{\partial BEI_i}\ with\ i\ being\ the\ tenor$$

The notionals required to hedge the delta exposures of the RDA are then determined per bucket, which are denoted by $'B'$.

$$notional\ interest\ rate\ swaps = B_i^N \ / \ U_j^N \ \ for\ every\ bucket\ i\ and\ bucket\ instrument\ j$$

$$notional\ inflation\ swaps = B_i^{BEI} \ / \ U_j^{BEI} \ \ for\ every\ bucket\ i\ and\ bucket\ instrument\ j$$

$$for\ i = j = 1,,7$$

If the resulting notional amounts are subsequently traded in the market, the portfolio is fully delta hedged on the bucket level.

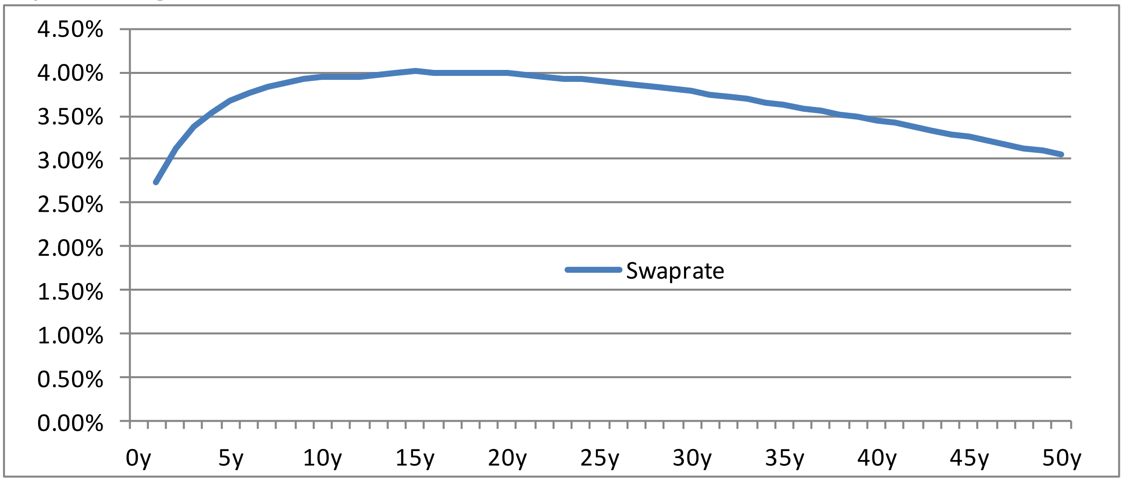
3.3. Dynamic hedging

A static hedge which is set up at the day the RDA premium is paid by the client works reasonably well in the first years, but loses hedge effectiveness over time causing a gap between the investment portfolio and the RDA value. This is also shown in the sensitivity analysis. Because the deltas of the RDA change over the lifetime, the hedge has to be adjusted accordingly. Yearly rebalancing is chosen in a tradeoff between costs and hedge effectiveness. This means that every year the swap positions are liquidated[14] against market value and that the proceeds or losses are added to the cash account, which is consequently deposited against the one year interest rate until the next rebalancing period. The deltas of the RDA are then recalculated and a new delta hedge is set up for both inflation and interest rates using the same buckets every period. All hedging positions thus last for one year and are then rebalanced. Section 3.5 will show step by step how this procedure is built into a simulation study. First section 3.4 describes the characteristics of the scenarios used in the simulation.

3.4. Scenario generation

The scenarios used in this paper are generated from the essentially affine term structure model described by van den Goorbergh, Molenaar, Steenbeek and Vlaar (2011). The input for this model consists of seven state variables which are generated using a quarterly vector autoregressive system with time-varying volatilities and correlations and stochastic jumps.[15] Since we only need yearly data we annualize the dataset so that we have 5000 scenarios of 30 years. The starting term structure is a long term equilibrium term structure based on the calibration period of 1973 to 2010. This term structure is shown in graph 1:

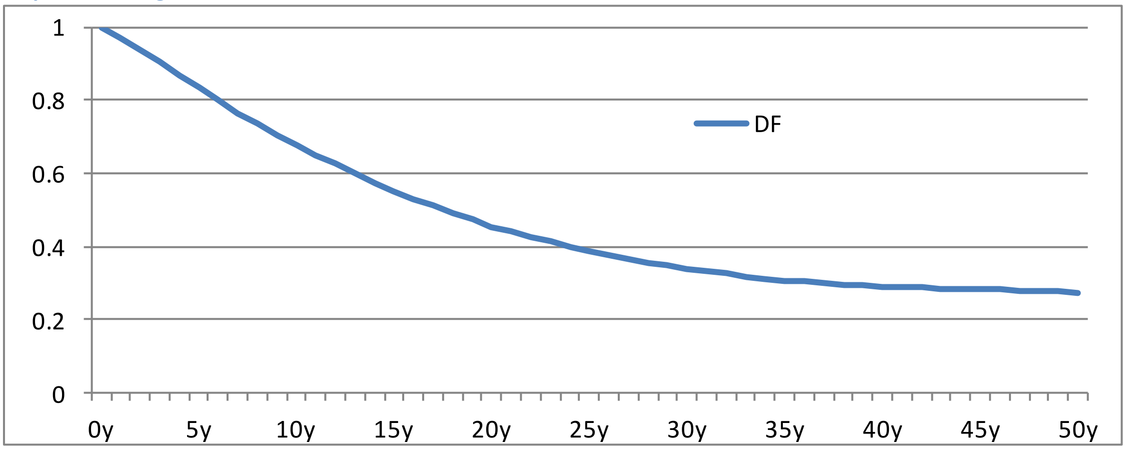
Graph 1: Starting interest rate term structure



Recall that the nominal swap rates are used to build the discount curve; graph 1 represents the Euro swap curve at the starting point of all the simulations. The swap rates up to a maturity of 50 years are shown as the 50 year swap is the longest maturity that can be traded. Bootstrapping[16] this swap curve gives us the discount curve shown in graph 2.

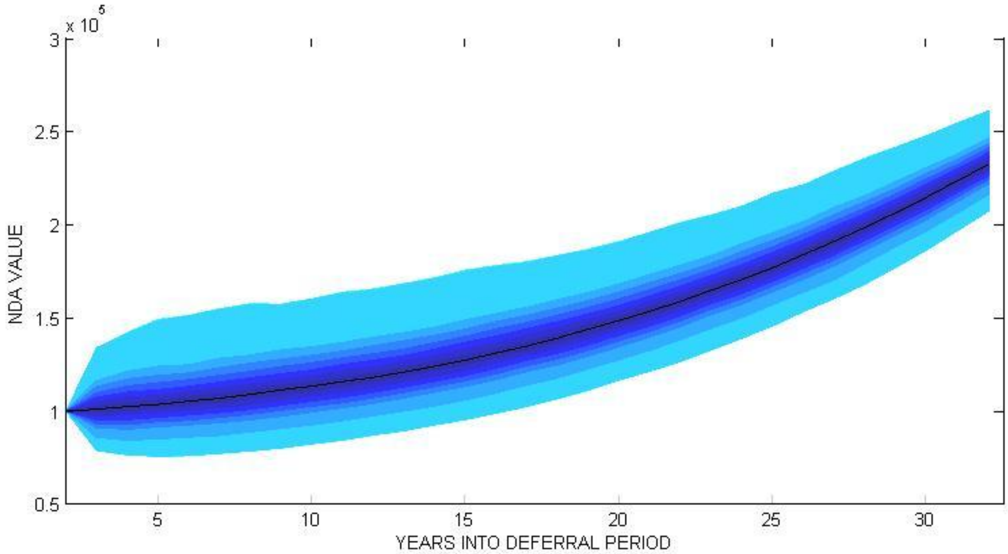
[14] Without taking transaction costs into account this is comparable with taking the delta of the existing swaps into account and only hedging the residual risk, which is done in practice. For simplicity this method is chosen.
[15] The scenarios of the state variables used are generated using a data generating process developed at APG.
[16] Bootstrapping is a method to build curves and is discussed in the appendix.

Graph 2: Starting nominal discount curve



Note that the discount curve is much steeper in the shorter term points compared to the longer maturities. This is referred to as curvature of convexity to which we will come back to in the result section. Each cash flow of the annuity is multiplied by the discount factor of the payout date to obtain the present value. This implies that the value of a deferred annuity should go up when it comes closer to the start date. Graph 3 shows that this is indeed the case for a 30 year deferred nominal annuity. [17]

Graph 3: 90% confidence interval of NDA value over time in 5000 scenarios



Graph 3 shows the NDA value over time in 5000 different scenarios of the interest rate. There is quite some dispersion but the average value at t=30 is between two and two and a half times that of the € 100,000 investment that our 40 year old client has reserved to buy a pension product. With this amount she intends to buy a deferred annuity that starts at 70 and pays out for 20 years.[18] Since this product is not available in the market against reasonable costs, she is looking for the best possible replicating portfolio. With this NDA, she would receive € 16,891 a year for a period of 20 years. Graph 3 also shows the conversion risk: the lump sum price for the nominal annuity at retirement

[17] This is a fan chart showing the 90% confidence interval. Every color level denotes a 5-percentile, while the median is depicted by a black line.

[18] In real life the client would be opting for a life annuity, but for simplicity a 20 year benefit period is taken. As stated earlier, we are trying to prove the concept of hedging a future stream of cash flows. This goal and the results will not change when mortality is introduced.

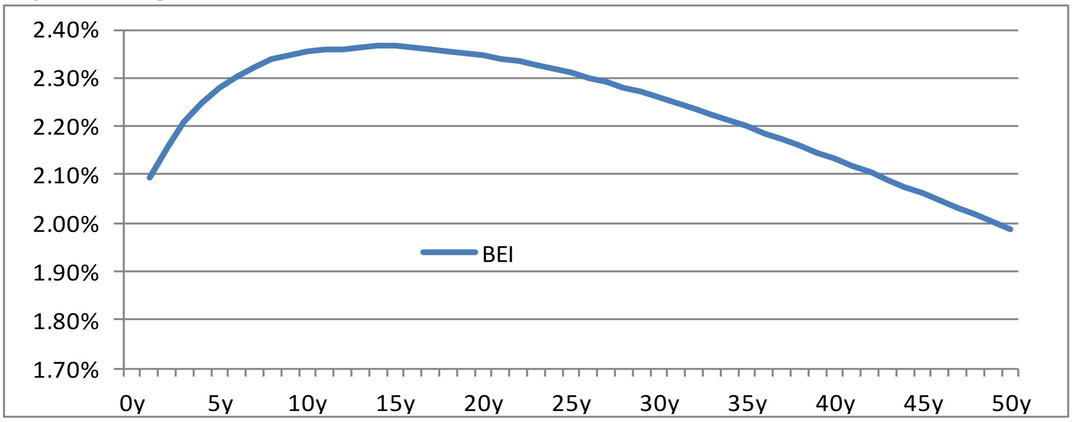
can be anywhere between € 206,000 and € 262,000 depending on the interest rate term structure at that moment, without counting the outliers. The RDA shows a slightly wider spread because the possible combination of low interest rates and high inflation or the opposite makes the tails fatter.

Table 2: NDA and RDA value descriptives

	NDA values		RDA values	
t	15 years	30 years	15 years	30 years
Average	€ 137,379	€ 232,829	€ 137,666	€ 223,452
Volatility	€ 23,836	€ 16,521	€ 20,154	€ 24,062
Skew	0.61	0.22	0.68	0.52
Kurtosis	0.76	0.06	0.87	0.37
5th percentile	€ 102,144	€ 206,698	€ 108,257	€ 187,971
25th percentile	€ 120,869	€ 221,401	€ 123,849	€ 206,122
Median	€ 134,823	€ 232,148	€ 135,406	€ 221,001
75th percentile	€ 151,582	€ 243,717	€ 149,407	€ 238,162
95th percentile	€ 180,315	€ 261,183	€ 173,905	€ 266,401

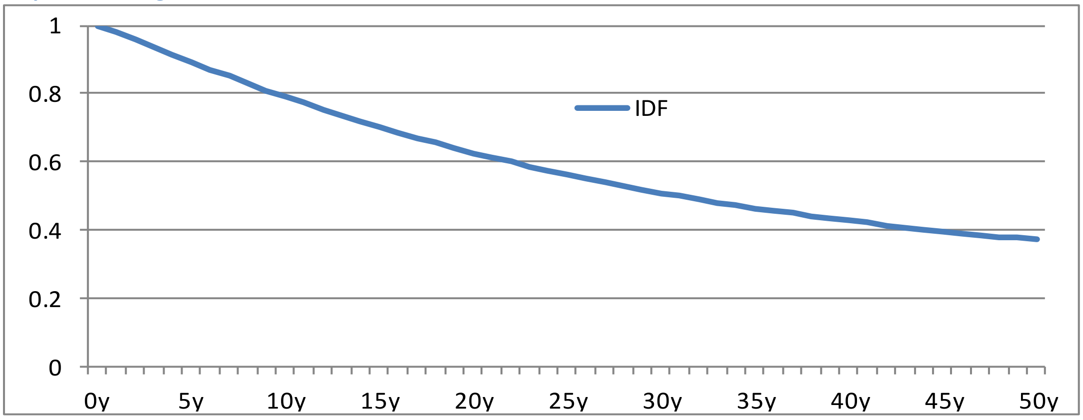
The purchasing power of the nominal amount will be highly uncertain as it depends on the inflation during the deferral period and afterwards during the payout phase. Graph 4 shows the initial break even inflation term structure.

Graph 4: Starting BEI term structure



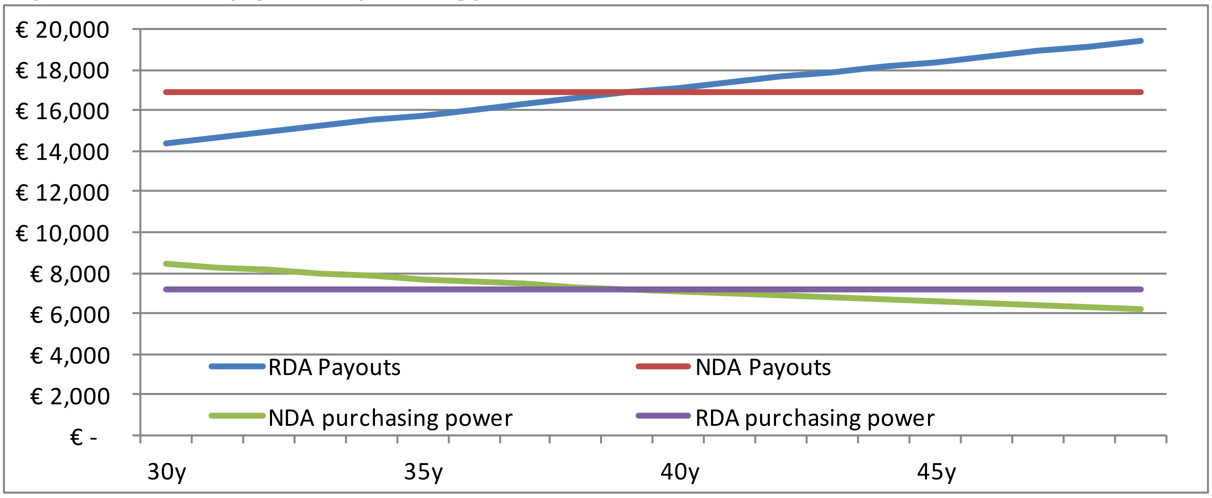
The 50 year BEI is approximately two percent, equal to the inflation goal of the European central bank. This curve translates into the inflation discount curve shown in graph 5.

Graph 5: Starting BEI discount curve



From this graph we can see the impact of inflation on the value of a deferred annuity. The 31 year[19] inflation discount factor is around the 0.5 mark, meaning that the purchasing power of the first annuity payout of the client is worth half the € 16891 in real terms and even less for the subsequent payouts. Note that this is only the case when the break even inflation realizes; it can be even worse if inflation shocks occur intermediately. To prevent getting harmed by inflation shocks and to have a better view on the future purchasing power, the client considers a real deferred annuity. Under the initial interest rate and BEI term structures, a RDA as defined above pays out € 7184 in *real* terms. On first sight, this looks much less than the nominal annuity and therefore a worse choice. However, the amount is stated in today's purchasing power. Given that the 31 year BEI discount factor is about 0.5, the expected nominal payoff should be divided by this number. We then get € 14420, which is much closer to the NDA payout. This amount is however the expected benefit at the first payout date. The second payment should be divided by the 32 year BEI discount factor and so on. The different payout patterns are shown in graph 6. It also shows the resulting purchasing power.

Graph 6: NDA and RDA payouts and purchasing power over time



The final RDA payment is almost € 20,000 while the nominal payment is still constant at € 16,891 by that time. The consequence in real terms is that the RDA facilitates consumption smoothing whereas the NDA provides declining purchasing power over time.

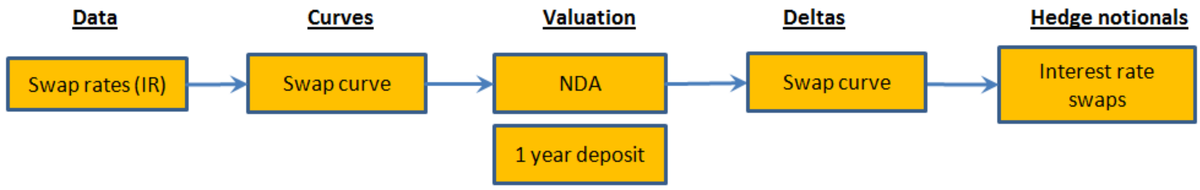
The sensitivities or deltas are hedged using the method discussed in section 3.2. How this approach is integrated in a 30 year period is explained next.

[19] We use the 31 year discount factor because the payments are at the end of the year. If we would have used start of year payments, the 30 year discount factor would be used.

3.5. Replicating portfolio

Suppose the client is attracted by the high nominal payouts the NDA offers and opts for a defined contribution solution in which the portfolio is designed to track the theoretical value of the NDA. The provider selling this product will set up the investment portfolio as depicted in figure 2. [20]

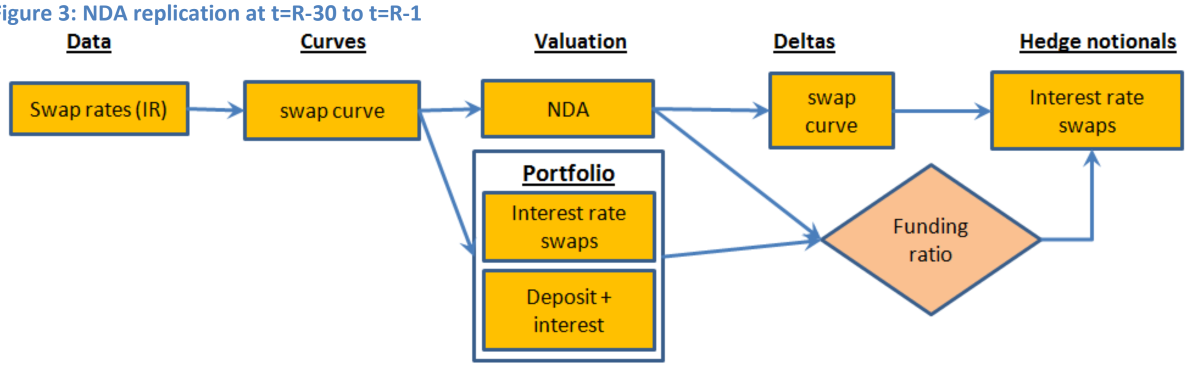
Figure 2: NDA replicating portfolio at t=0



At initiation, the swap rates are used to build a swap curve which is subsequently used to value the NDA. The deltas of the NDA are then calculated so that the hedge notionals can be determined. The investment of the client is put in a 1 year deposit.

The first step is using the swap curve to value the NDA following the bootstrapping methodology described in the appendix. Since the value of the NDA is sensitive to the swap curve, the deltas are determined and bucketed as described in section 3.2. These deltas subsequently determine the notionals for the swaps. These swaps will be kept in the portfolio for one year while the investment from the client is deposited against the one year interest rate. The process after one year is then:

Figure 3: NDA replication at t=R-30 to t=R-1



At the start of every year from t=1 onwards, the NDA and the swap portfolio are valued using the swap curve and the latter is added to the one-year deposit. Next, the NDA deltas are determined so that the swap notionals can be calculated, corrected for the current funding ratio.

After one year the NDA is revalued given the current term structure and the shift in time. The latter refers to the fact that the first payout is only 29 years from t=1 such that the stream of cash flows shifted up the discount curve. The value of the portfolio has changed with the accrued interest from the deposit and the marked-to-market value and reset payments of the interest rate swaps. The funding ratio is then determined by dividing the portfolio value by the theoretical value of the NDA. Recall from section 2 that the funding ratio will be equal to one if

$$\Delta PV_t(Portfolio) = \Delta PV_t(NDA)$$

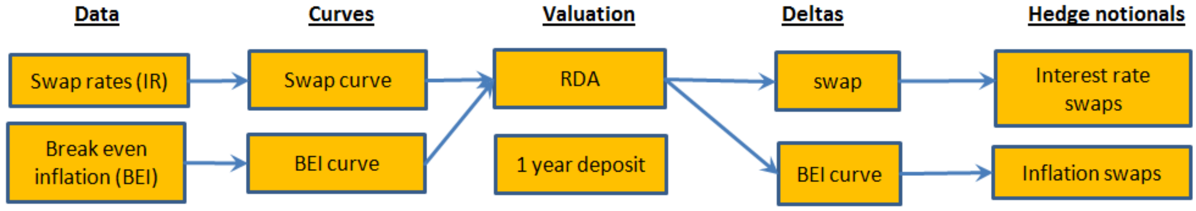
The results will show that the hedge is not perfect, so the above equation only approximately holds.

[20] The whole case is treated as if the client is the only customer of the provider. In practice, the provider would add the money received from the client to its assets and add the resulting liabilities to its large pool, and hedge the total risk.

For the next year, the hedge is set up similarly as at t=0, but now the notionals are adjusted for the funding ratio. The resulting hedge is referred to as a funding ratio hedge. If the current funding ratio is ignored, a surplus hedge is put in place. This has the consequence of overhedging when the FR falls below one and underhedging when the FR is more than 100%. Since keeping the funding ratio stable over time is the goal of the hedge, the FR hedging approach is used. In the results section the consequences of picking one of the two approaches are highlighted. The process in figure 3 is repeated until one year before retirement, when the last hedge is put in place. At retirement, the complete portfolio is liquidated and paid out to the client so that she can buy a nominal annuity without conversion risk, as this risk was hedged with the swaps during the deferral period.

Given that the client chooses the RDA as a DC pension provision, the provider has to manage the € 100000 such that its tracks the value of the RDA as good as possible. Figure 4 describes how the initial hedge is set up. First, the current interest rate and inflation term structures are used to build the required curves following the bootstrapping methodology. The curves are subsequently used to determine the yearly payout the client would receive under the current term structure based on the size of the investment. This results in a stream of real cash flows which is dependent upon the future interest rate and inflation. The sensitivity is quantified and put into buckets as described in section 3.2. The bucket deltas are then hedged using the respective bucket instruments.

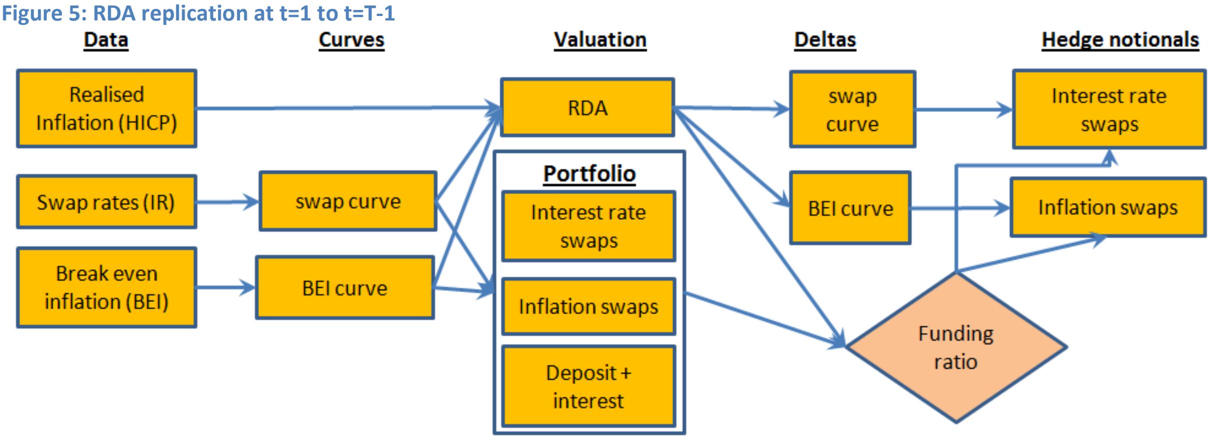
Figure 4: RDA replication at t=0



At initiation, the swap and BEI rates are used to build a swap and BEI curve which are subsequently used to value the RDA. The deltas of the RDA with respect to both curves are then calculated so that the hedge notionals can be determined. The investment of the client is put in a 1 year deposit.

The bucket instruments are traded with the calculated notionals and will stay in place for one year. At the end of the year, the portfolio is revalued and the hedge rebalanced. Figure 5 shows this process.

Figure 5: RDA replication at t=1 to t=T-1

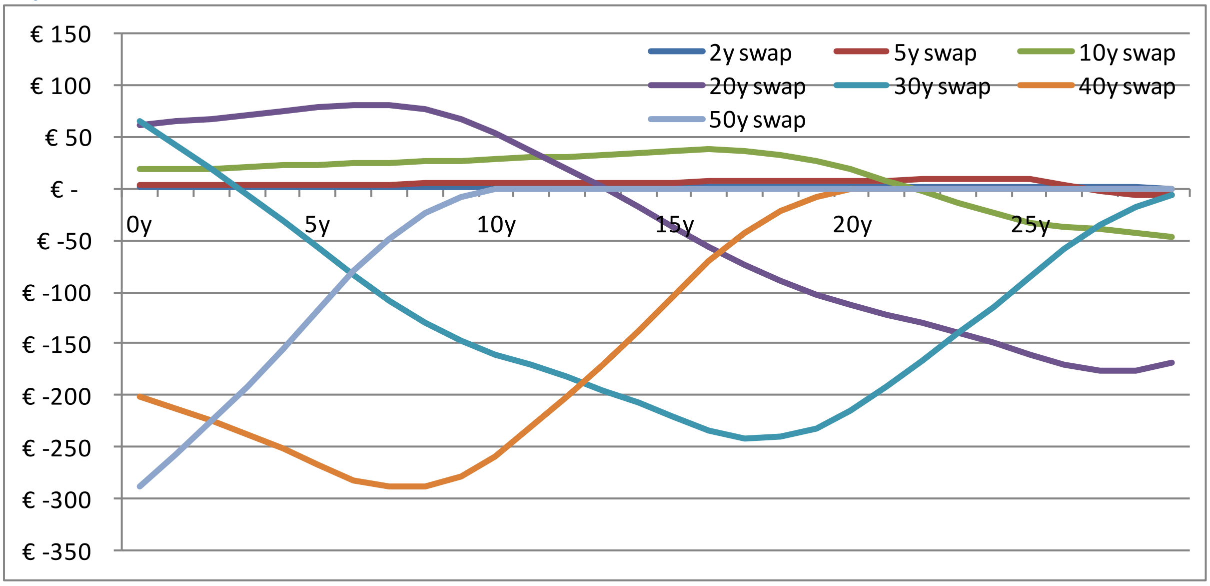


At the start of every year from t=1 onwards, the swap portfolio is valued using the swap and BEI curve and added to the one-year deposit. To value the RDA, the accrued inflation is also taken into account. Next, the RDA deltas are determined so that the swap notionals for the next year can be calculated, corrected for the current funding ratio.

Firstly, the RDA is valued using the current swap and BEI curves, taking the accrued inflation into account.[21] The portfolio value is the sum of the deposit and its accrued interest and the marked-to-market value of the hedges, including the reset payments. The hedge for the next period is corrected for the current funding ratio analogous to the NDA process.

As an example of how the hedges are structured throughout the deferral period, graph 7 shows the RDA interest rate bucket deltas over time.[22] The term structures are held constant at the initial levels to isolate the time effect from curve movements and the realized inflation is two percent a year. The delta values on the y axis are in Euros, based on the € 100,000 investment of the client. The value indicates what happens to the value of the RDA when the particular swap rate goes up by one basis point. At initiation, the sensitivity to the 50 year swap rate is such that if it falls by one percent, the value of the RDA goes up with almost 30 percent. This illustrates the need for hedging interest rate risk.

Graph 7: RDA interest rate deltas over time under constant interest and BEI rates

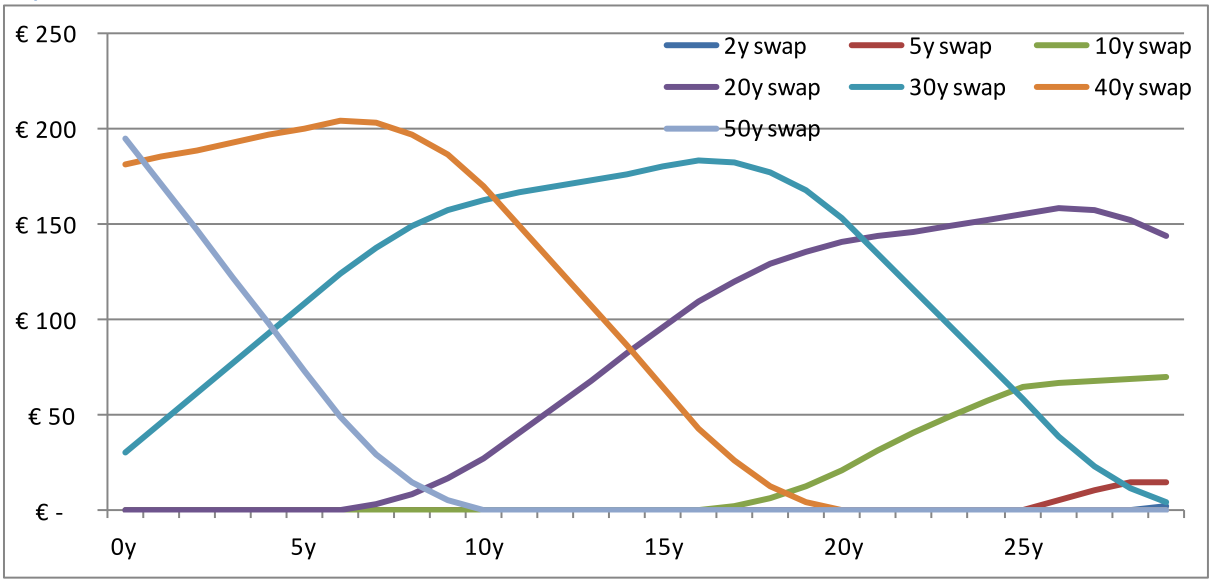


Contrary to what one might expect, the deltas are not exclusively negative which would indicate that any rise in the term structure lowers the present value of the RDA. This is due to the sequential process used to build the curve, as explained in the appendix. The high maturity discount factors depend upon the low maturity ones causing a non-linear relationship between swap rates and the resulting discount factors. The negative deltas are hedged with receiver interest rate swaps which pay a fixed rate in exchange for a floating rate. As a result, the provider has to pay less when the interest rate goes down while receiving the same amount annually. This compensates for the increased value of the RDA. The positive deltas are hedged with payer swaps, which have the opposite effect from the receiver swaps. This way the swap curve build-up effect is hedged.

[21] As discussed in the appendix, there is a lag of 2-3 months between the realization of inflation and the actual reporting of the number by the statistical agencies. In the simulations we don't suffer from this problem because the realized inflation scenarios are already generated. For simplicity the lag is therefore ignored.
[22] The picture for the NDA shows a similar pattern over time, but because it is less backloaded than the RDA the middle maturity deltas go up at the cost of the high maturities.

Graph 8 shows the more intuitive view of the RDA inflation deltas for which the curve is derived from zero coupon instruments. The relationship between the BEI rates and the resulting inflation discount factors is linear resulting in solely positive deltas:

Graph 8: RDA BEI deltas over time under constant interest and BEI rates



The positive deltas indicate that the RDA becomes more expensive when expected inflation rises. This effect is hedged with payer zero coupon inflation swaps which pay out the realized inflation in exchange for the contracted break even inflation rate.

Note that the long term hedges decline in mass over time giving room to the shorter term hedges which are needed when the cash flows come closer to maturity. In this individual case, not all buckets are used simultaneously but only when the cash flow stream is within range. For a provider with a large pool of clients these lines will be more or less horizontal, as new clients enter and mature clients leave the aggregate deltas will be relatively stable through time. This might lower transaction costs because there is less need for rebalancing. This could be a vital part of keeping the costs down, even though the costs per swap are quite low.

This chapter has outlined the simulation methodology supported by tools discussed in detail in the appendix. How this toolbox performs in a simulation study is presented next.

4. Simulation results

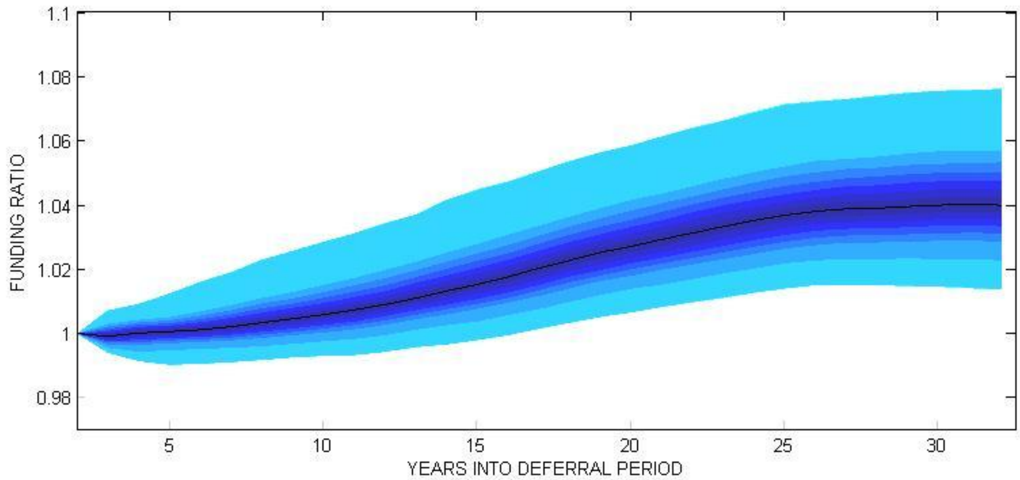
To test which investment strategy performs best in replicating the RDA, we will simulate all different options over 5000 scenarios with stochastic state variables. We start with replicating a nominal annuity and expand to the real case by including inflation next. A simulation with a traditional portfolio of stocks and bonds is done for comparison.

The numerical results will be presented in two ways: A table containing the descriptive statistics and a graph. The graph type is a fan chart which shows the 90 percent confidence interval divided in 18 five-percentile buckets which can be distinguished by brightness.

4.1. NDA simulation results

Graph 9 shows the distribution of the funding ratio over 5000 scenarios during the deferral period.

Graph 9: 90% confidence interval of funding ratio of FR hedged NDA



The hedge performs quite well, with the FR never falling below 98 percent and at retirement not even below 100 percent in the 90% confidence interval. This indicates that a nominal guarantee is a promise the provider can actually make and hold, regardless of the interest rate term structure at that moment. As transaction costs and fees are not taken into account here, the guarantee level might have to be set somewhat lower in the real world, but there is a clear floor. It may seem peculiar that the funding ratio rises over time instead of remaining constant at the 100% mark. This is due to small embedded duration bets that are still present in the hedge. Interest rate risk is therefore not completely eliminated, but limited to a small probability (0.6%) of the final FR being slightly below one. This risk is rewarded with a risk premium which pushed the expected final FR slightly above one. The effect tops off in the end of the deferral period, because the risk premium becomes smaller with the decreasing duration of the liabilities and the hedge.

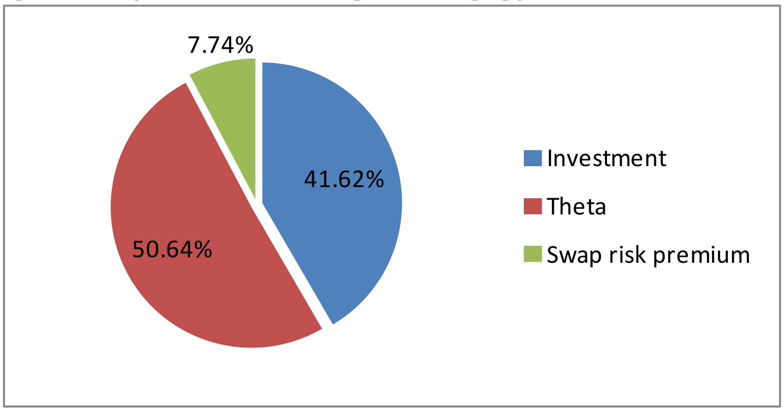
The descriptive statistics of the nominal simulations are shown in table 3 on the next page. Even though the average FR at maturity is higher for the funding ratio corrected hedge, the volatility is not. This goes against theoretical assumptions and against common sense. A surplus hedge solely aims at preserving an absolute amount of money while a funding ratio correction aims at preserving the relative FR measure. The extra volatility is not evenly distributed; the lower bound of the FR corrected hedge is higher but the upper bound outperforms the surplus hedge with a larger difference. This is visible in the 5-percentiles in table 3. Even though volatility minimization is a part of the goal when giving a guarantee, we let the theoretical considerations and the higher bounds prevail and continue with only FR corrected hedges. Another argument for this is that the FR hedging approach makes more a difference when the funding ratio is far away from one, which barely occurs in this simulation.

Table 3: NDA hedge portfolio descriptives

	Surplus hedged		FR hedged	
t	**15 years**	**30 years**	**15 years**	**30 years**
Average	101.91%	103.22%	102.23%	104.19%
Volatility	1.55%	1.76%	1.55%	1.95%
Skew	1.34	0.89	1.03	0.70
Kurtosis	3.73	1.95	2.49	1.29
5th percentile	99.91%	100.74%	100.15%	101.36%
25th percentile	100.86%	102.01%	101.17%	102.85%
Median	101.67%	103.03%	102.03%	103.98%
75th percentile	102.65%	104.20%	103.03%	105.31%
95th percentile	104.69%	106.36%	105.03%	107.59%

To illustrate how the final funding ratio is built up, figure 6 shows that the initial investment is only 42% of the final value. As graph 3 shows, the NDA value more than doubles when the cash flows move up the discount curve over time, giving the time effect or ‘theta’ a large weight. The cumulative risk premium amounts to almost 8 percent of the final portfolio value.

Figure 6: Components of final average NDA hedging portfolio

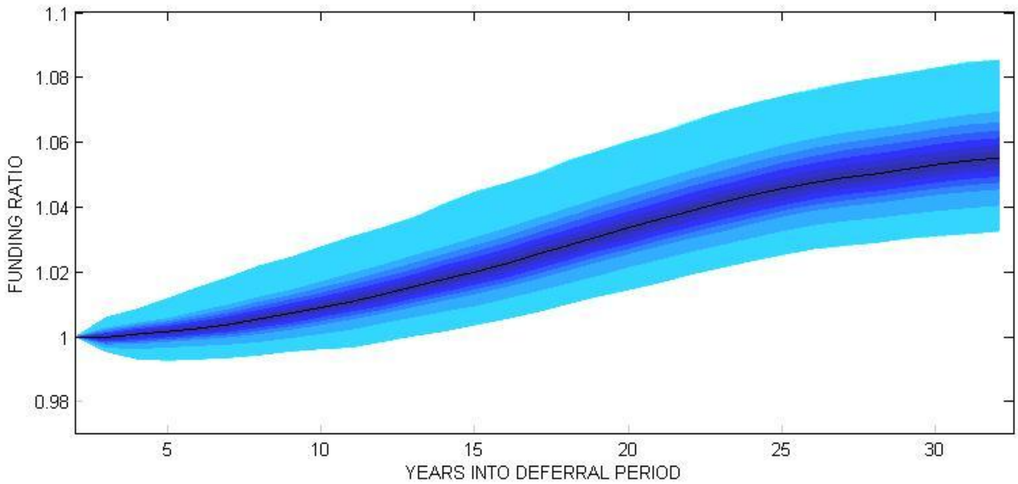


Regardless of the surprising performance of the surplus hedge, the aggregate result is quite satisfying and workable. However, the real deferred annuity is the objective with the nominal part a means to get there, not a goal in itself. The simulation results for the real framework are presented next.

4.2. RDA simulation results

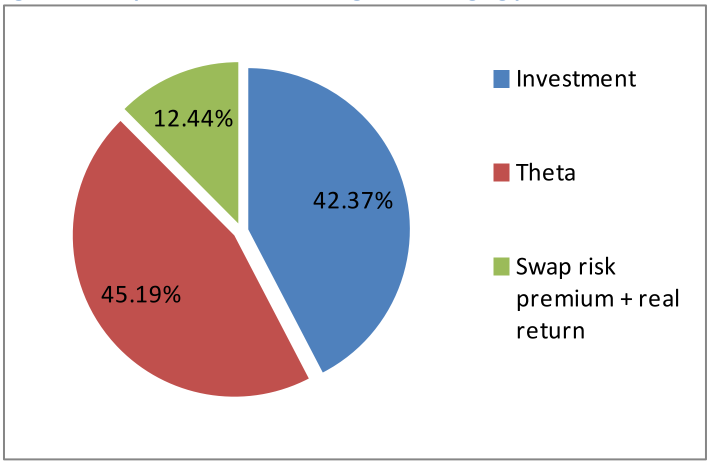
Graph 10 shows that the replicating portfolio using interest rate and inflation swaps is capable of providing certainty during the accumulation phase:

Graph 10: 90% confidence interval of funding ratio of FR hedged RDA (7 buckets)



As table 4 also shows, the expectation is that the final FR will be between 103 and 108 percent. The variation here is caused by differences in real returns over the scenarios combined with the varying swap risk premiums as with the NDA. The real returns are earned by the spread between the nominal rates derived from the swap curve and the BEI rates. There are cases in which this return is negative, but this is usually temporary and when that happens the effect tends to be dominated by the swap risk premium discussed earlier. Figure 7 shows that the return is larger for the RDA when compared to the NDA. The first reason for this is the backloading of the RDA which increases the duration and with that the earned risk premium, the second reason is the real return. Contrary to the NDA case, the lowest observation of the final RDA FR is above one, which is due to the addition of the real return.

Figure 7: Components of final average RDA hedging portfolio



The risk premium effect is also present in the inflation swaps, but it is much smaller because the BEI rates are not so far apart for different tenors as the swap rates. (compare graphs 1 and 4)

The outcomes are slightly different when fewer buckets are used, which is discussed next.

Table 4: RDA hedge performance descriptives

	3 buckets		5 buckets		7 buckets	
t	**15 years**	**30 years**	**15 years**	**30 years**	**15 years**	**30 years**
Average	101.72%	102.06%	103.73%	107.78%	102.66%	105.64%
Volatility	2.16%	2.50%	1.49%	1.91%	1.34%	1.65%
Skew	0.46	0.45	0.41	0.39	0.74	0.57
Kurtosis	0.87	0.70	0.83	0.49	1.65	0.97
5th percentile	98.44%	98.35%	101.45%	104.87%	100.75%	103.23%
25th percentile	100.27%	100.37%	102.75%	106.49%	101.77%	104.53%
Median	101.59%	101.92%	103.64%	107.67%	102.53%	105.50%
75th percentile	103.00%	103.56%	104.61%	108.94%	103.41%	106.59%
95th percentile	105.54%	106.45%	106.32%	111.19%	105.03%	108.51%

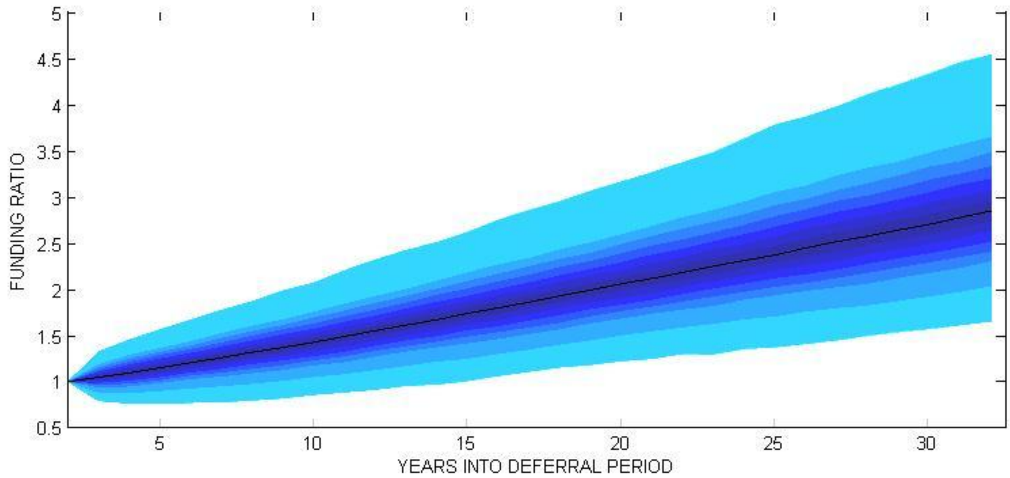
Table 5 also shows the differences between hedging with less than 7 buckets. We can conclude that using more buckets reduces potential bucket mismatch resulting in lower volatility. However, the returns for the five bucket approach are higher while three buckets do worse. This is caused by different duration bets embedded in these strategies. Recall from graph 1 that the equilibrium term structure is upward sloping until 15 to 20 years maturity and downward sloping afterwards. As can be seen in appendix 7.4, the five buckets approach places the short term deltas in the bucket which is hedged with the 10 year instrument. In effect, short term RDA sensitivities are hedged with longer term instruments (with higher rates) which generates a profit on average but makes the hedge more sensitive for changes in the shape of the curve. The three buckets approach is even more exposed to this, but places the 15 and 20 year deltas into the 30 year bucket and the 35 and 40 year deltas in the 50 year bucket. Since the long end of the curve slopes downwards, the reverse of the five buckets case happens. Hedging is done with an instrument which rate is lower than that of the RDA part it is hedging, and therefore loses in the long run *and* incurs more volatility, pushing the lower bound below the 100% mark. This is the reason why seven buckets are chosen. One could go further in this, but the illiquidity of for example the 45 year instrument and the extra costs make this less feasible in practice, while differentiating further in the liquid short end of the curve will not improve the efficiency much more because the RDA sensitivity is gravitated towards the long end of the curve.[23]

As a side note we can conclude from this that a DB pension fund that does not hedge its liabilities but invests in short and middle term bonds effectively takes a duration bet that has a negative expected return in the long run, if one considers an upward sloping curve as equilibrium. This is caused by the difference in the short term interest it earns on the bonds against the long term interest rates against which the liabilities move up the discount curve over time. This duration mismatch is a potential threat to the sustainability of DB schemes which have lost a lot of value during the persistent interest rate decline in the past 20 years. A DC scheme does not suffer from this problem, but that is only because it has no liabilities and puts all risk on the participant. How such a scheme performs in tracking the RDA is discussed next.

[23] This story is analogous for the inflation hedge, but the effect is much smaller because the difference in BEI rates is much smaller for different tenors.

The DC portfolio has 50% in a bond portfolio (Barclays aggregate short term) and 50% in stocks (MSCI world) and is fully hedged for currency fluctuations. Equivalent to the hedging solutions, it is rebalanced yearly.

Graph 11: 90% confidence interval of RDA funding ratio in traditional DC



This graph suggests that DC outperforms the hedging approach in every scenario, which is probably a reason why it is so widespread. The lowest outlier is around a 63% funding ratio which can be considered a tail event, but tail realizations are not uncommon in recent history. Also, the costs are not taken into account here. Recall from section 2 that asset management costs of British DC schemes have been reported to be around 37% of final portfolio value, slashing the expected gains and turning the tail events into bankruptcy for the client. It has to be said that with today's index tracker funds it must be possible to get this portfolio against lower cost though. Contrary to the scenarios which involve hedging, this asset management simulation is heavily dependent on return assumptions[24]. The (arithmetic) expected return of the equity and bonds in this simulation is 8.25% and 4.19% respectively, which can be considered too optimistic. This results in the probability of shortfall being very low and not visible in the 90% confidence interval in graph 11. De Jong et al. (2008) use more prudent assumptions resulting in an 8.4% probability of falling short of the guarantee with an average shortfall of 14.6% of the total indexed contributions.[25]

Table 5: Stocks and bonds as RDA hedging portfolio descriptives

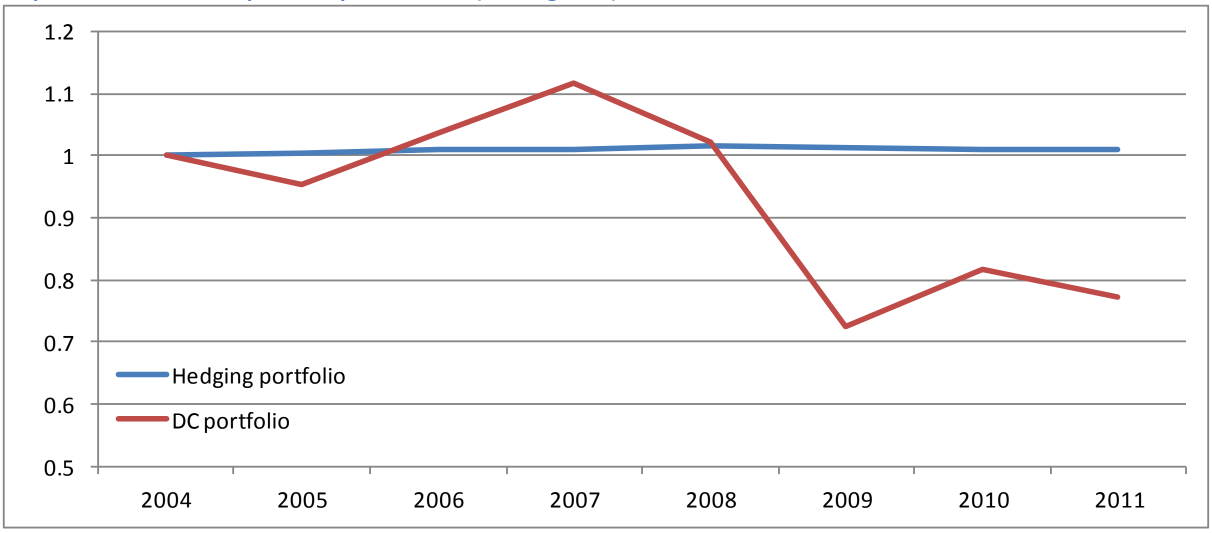
t	15 years	30 years
Average	190.29%	293.74%
Volatility	53.74%	89.70%
Skew	0.49	0.66
Kurtosis	0.32	0.68
5th percentile	110.57%	164.83%
25th percentile	151.51%	230.22%
Median	186.01%	284.40%
75th percentile	224.77%	347.78%
95th percentile	285.67%	453.40%

[24] The RDA hedging approach is not entirely free of this, a high real return will lead to a higher FR.
[25] The deferral period is the same, but de Jong et al. (2008) use a different benefit tenor. It is therefore not perfectly comparable but the main point is that the return assumptions are very influential.

The main point is however that regardless of the assumptions, the client has to accept a non-zero probability of his assets falling very short to cover the liabilities. If one cannot bear such an event, any non-zero probability is too high. For excessive wealth this is an attractive option, but for a low income earner without downside endurance the term 'Lifetime ruin minimization' used by Huang and Milevsky (2011) describes the situation more accurately, even though it is pulled out of context here. The consequences of showing a picture like the above can be quite devastating, because it looks like nothing can go wrong. In the United States it is common to use a liability discount rate of 8% to establish premium levels (Biggs, 2010). This seems very reasonable when the expected return is accordingly, but this assumes that the pension benefit payouts are also subject to stock market volatility. As discussed in the introduction, this lack of guarantee is sub-optimal for the welfare of retirees.

To see how the portfolios compare in a tail event, all we have to do is to look at recent history. Suppose the client is a 60 year old man who, in 2004, decides to reserve € 100000 of his wealth for retirement provision. The effects of which investment portfolio he puts it in can be quite severe; taking the end-of-year values of stocks, bonds, HICP and interest and BEI rates results in the crisis scenario below.

Graph 12: Crisis scenario portfolio performance (funding ratio)



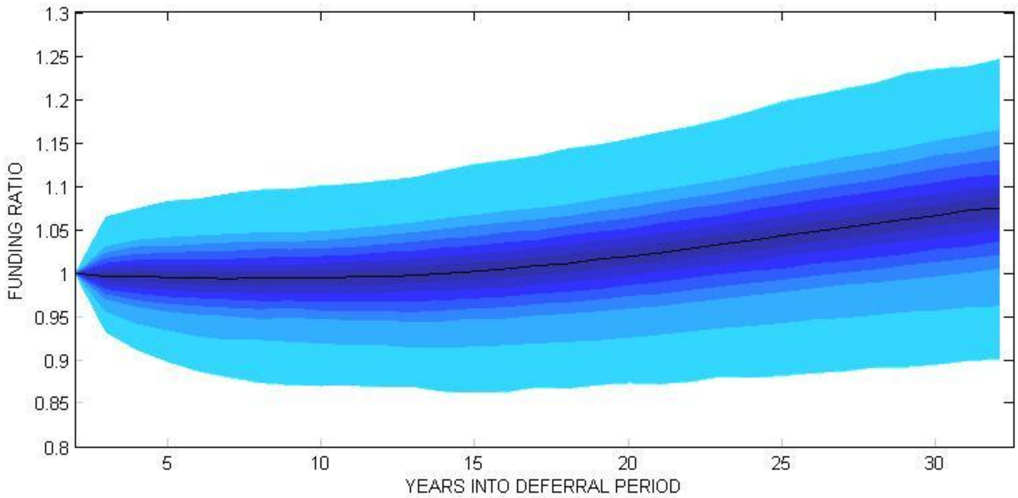
During the credit crunch both stocks and interest rates plummeted, which increased the bond portfolio but destroyed the equity wealth. The rise in bonds cannot compensate for the increase in the value of the liabilities of the 20 year annuity which the client planned to buy at the beginning of 2012, because of the duration gap between the two sides of the balance sheet. This leads to a 23% lower lifetime income from the bought annuity. Somebody who decided to step in around April 2007 will be even worse off[26]. The advantage of the hedging portfolio is obvious, the funding ratio barely moves during this rough period. A side note that has to be made is that the hedging portfolio is more susceptible to counter party credit risk, which is considered at the end of this chapter.

To segregate the effects of the two types of hedging instruments used, graph 13 shows the funding ratio when the RDA is only hedged with interest rate swaps, leaving the inflation risk open. This

[26] We would have liked to take a wider timeframe, but the long term BEI rates were not quoted before 2004. With the dot-com crisis taken into account the results would likely be even more extreme.

might be a viable option when inflation swaps are expensive or hard to trade for liquidity reasons, or simply as an investment choice. Graph 13 shows that the dispersion around the median is much larger in this case.

Graph 13: 90% confidence interval of nominally hedged RDA



Given the limited risk displayed, a case can be made for not hedging inflation risk. This does open up vulnerability to a tail event such as what is happening in Great Britain currently, where inflation levels are nearing 5% while the central bank keeps interest rates low. While the lowest observation of the fully hedged RDA FR is still above 100%, the minimum observed FR without inflation protection is 71% which lies in the first percentile which is below 84%. From this perspective the downward risk is severe, which questions nominal regulation of pension funds in general.

Table 6: Nominally hedged RDA descriptives

t	15 years	30 years
Average	100.54%	107.48%
Volatility	8.10%	10.48%
Skew	-0.17	0.00
Kurtosis	-0.09	-0.08
5th percentile	86.91%	90.11%
25th percentile	95.18%	100.51%
Median	100.88%	107.47%
75th percentile	106.17%	114.66%
95th percentile	113.45%	124.53%

The importance of indexation can be denoted in a simple observation: the average cumulative inflation after the 30 year deferral period is about 81% with a standard deviation of 17.5%, which implies a risk with a rather large magnitude. For people without a financial buffer, not insuring against a tail event here is more or less equivalent to not taking fire insurance on their house. In expectation it is financially better not to, but the cost of the event when it actually takes place is simply too high.

4.3. Sensitivity analysis and limitations

To achieve the results the following parameters were chosen:

1. Hedging is primarily done using 7 buckets
2. The deferral period is 30 years and the benefit period 20 years
3. The hedging portfolio is rebalanced yearly

The different results from using different amounts of buckets have already been discussed in the previous section. Because bucket hedging carries the assumption of parallel movements within the buckets, using more buckets weakens this assumption and this turns out to work better.

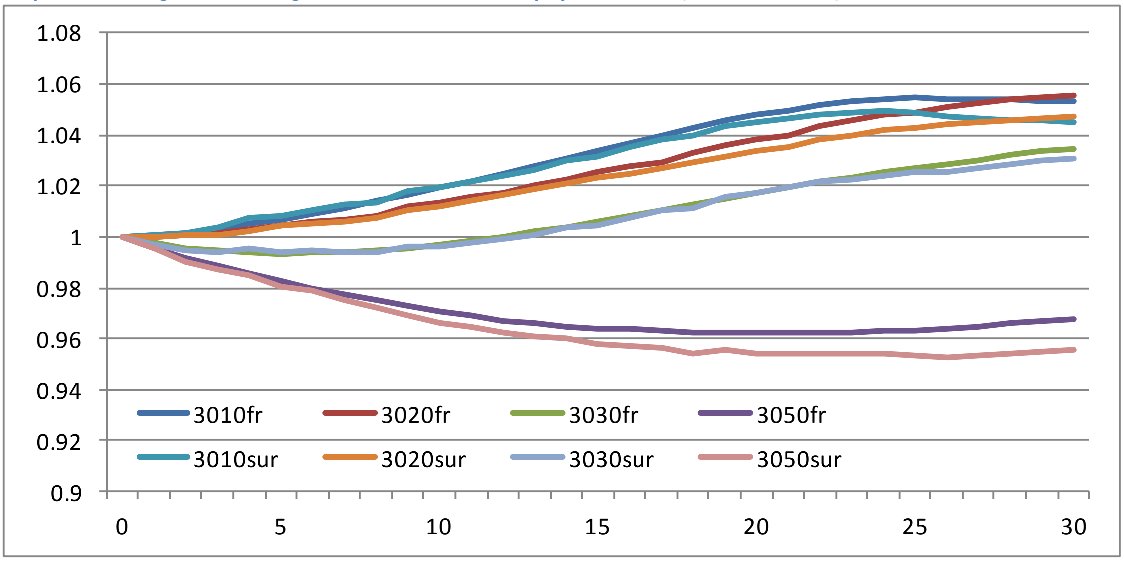
The second point concerns the consequences of picking different tenors, which include convexity and theta effects. This is discussed in the next subsection, while the rebalancing period is treated afterwards. Section 4.3.3 lists the limitations and discusses after which section 4.3.4 elaborates on some issues concerning practical implementation

4.3.1. Convexity & theta

The RDA is delta hedged, but the delta is a linear approximation that works well for small changes but can be partly off when big shocks occur. Since we are (justifiably) using a model that has jumps incorporated in it, convexity can have effects on the aggregate results. Hull (2007) states about this: 'The convexity of a bond portfolio tends to be greatest when the portfolio provides payments evenly over a long period of time.' Given that a deferred annuity provides a long stream of high duration cash flows, the convexity effects are maximized.

Recall from graphs 2 and 5 that the zero and BEI discount curves are not linear and thus have different curvature on different maturities. When the cash flows move up the discount curves as they come closer over time, the value of the RDA changes even when the rates do not change. This is referred to as the time effect or "theta'. The swaps used for hedging also change in value, but the theta effects need not be equal to that of the RDA. This can create mismatches. The differences in curvature on different tenors of the discount curves also cause different convexity effects. This is illustrated in graph 14, which shows four hedged RDAs, each with a 30 year deferral period but with different benefit durations. For every variant the surplus and funding ratio hedged FR is shown.

Graph 14: Average FRs of hedged RDAs with different payout tenors (1000 scenarios)



The bottom line shows the FR of a hedged RDA with 80 year total duration and never recovers above 98% FR after falling to 96%. The 40 year duration FR on the top rises almost the full time series, while the 30 year deferral into 30 year benefits RDA sticks just below a FR of 100% before rising. From the patterns above we can conclude that for maturities over 50 years the FR goes down, which is probably attributable to the extrapolation of the curve from the 50 year point onwards, leading to higher order effects for which we have no clear explanation. Contrary to the interpolation method, there is no obviously superior extrapolation method known to the author that would solve this problem. Bianchetti (2010) stresses that picking market instruments and bootstrapping is 'much more a matter of art than of science, because there is not a unique financially sound recipe for selecting the bootstrapping instruments and rules.' Ideally we would pick the classic 40 into 20 year structure or take the future into account and use something like 50 years deferral and 30 year benefits. This is left for further research which should use a more advanced extrapolation method than the flat (in the zero rate) used here. We therefore limit ourselves to the 30 into 20 case, which provides sufficient information about the conceptual achievability. The extrapolation issue is left as a trader's problem. Notable is that the hedges that are corrected for the FR outperform the surplus hedges at any tenor.

4.3.2. Rebalancing frequency

Because yearly rebalancing works quite satisfactory there is no reason to increase the frequency. This would impose extra costs and complexity due to inflation seasonality effects that have to be taken into account in the valuation and hedge setup. The benefits of using seven buckets do come at the cost of trading 14 swaps every year, of which the cost is calculated as a number of basis points of the swap notional. Because the swap deltas per unit are lower than that of the RDA, the notional has to be increased to match the sensitivities of both sides of the balance sheet. This balloons the sum of the swap notionals needed for the initial hedge to 8 times the investment the client put in. The fees are calculated over this number, which has a leveraging effect on the costs as percentage of the investment. This is why it might be worthwhile to rebalance less if the hedge effectiveness does not suffer too much. When the rebalancing frequency is halved and thus every two year rebalancing takes place, the average final funding ratio barely changes. However, the standard deviation over the scenarios doubles due to increased hedging mismatches and the observation that 2-year shocks are often larger than 1-year shocks, increasing convexity and theta effects. The lower bound of the funding ratio is pushed below a 100%. Net of costs less rebalancing might be marginally better, but in practice rebalancing is done more often because premiums flow in every month and a trade is done when a large enough volume is reached. Possibly rebalancing less than once a year is therefore not a likely strategy and is discarded.

4.3.3. Limitations

To achieve the results above, a couple simplifying assumptions were made. This section lists these limitations and discusses the effect on the results.

1. Linear interpolation is not the best practice for curve construction, as described by Hagan and West (2006). Because the valuation and hedging is done with respect to the same curve, a more complex method would not change the conclusion but would lead to complexity and slower simulations due to curve fitting. This is why the simplest method is applied, but a smoothing approach is advisable in practice.

2. A convenient side effect of rebalancing and paying out yearly is that the seasonality effects of inflation drop out of the BEI curve building and inflation swap valuation process. In practice monthly benefit payments are the standard, but the indexation is usually performed on a yearly basis, so this assumption should not affect the results.

3. Not including costs will affect the quantitative results, but can be adjusted for with the rule of thumb of multiplying the yearly fee by the duration of the product. Since cost levels are not established here this is left open.

4. Leaving out mortality probabilities was necessary to narrow down the problem to the testable hypothesis of being able to hedge a stream of real cash flows. On the individual level, the RDA as described in this paper is what a client might be looking for in the DC space regarding the accumulation phase. Conversion risk with respect to mortality is left open but currently it is very hard to hedge, and it can be questioned whether one wants this risk during the accumulation phase given the big risk premium that has to be paid. The hedging performance is unaffected, as mortality probabilities would simply lead to another stream of cash flows that would be hedged using the same method. There would be a quantitative effect, because the duration of the mortality corrected cash flows will be lower, decreasing the risk premium of the small embedded duration bets. The final FR will therefore be somewhat lower. Interesting future research can be done in by including mortality probabilities and costs in a monthly framework, so that indicative premium levels can be established, attaching a price to DB.

5. It is possible that hedge performance can be increased by using swaptions to hedge gamma effects in both inflation and interest rate movements. However, the possible setbacks are numerous: more complexity, higher costs, higher probability of operational errors to name a few. Most importantly, the results do not provide a motive for more advanced downside protection so that we can leave it at this.

6. Mismatches between the European HICP and the actual consumption pattern cannot be avoided, but the long term effects are expected to be limited as discussed in section 2. Given that we use BEI (which includes a risk premium) as expected inflation measure, it is more likely that the indexation of the benefits is too high rather than too low. In fact, we can calculate this mismatch: recall from graph 5 that the 31 year inflation discount factor was around 0.5, which would put the expected index value after 30 years at 200. The actual

average index value after 30 years is 181, indicating that BEI overstates the cumulative inflation by about 24% on a 30 year period. We conveniently consider this to be prudent rather than a problem, but in a competitive market there is a danger that a product priced this way will price itself out of the market. This can be solved by either using another measure for expected inflation or in the communication to the consumer, by stressing that this measure is closer to wage inflation than a more accurate alternative. (using 3% yearly wage inflation results in an index value of 242 after 30 years) This is however left for further research.

4.3.3.a. Collateral

The limitation of not incorporating collateral in the simulation is treated separately because of the large impact in has on the actual execution of the RDA replicating strategy. In this approach, derivatives are used to hedge financial risk by signing a contract with a counterparty which is not risk free itself. Hedging using derivatives transfers financial risk to counterparty risk, which means that the insurance against the financial risk is not worth anything if the counterparty defaults before payout date. This could be the case when interest rates have gone down and realized inflation is higher than expected, so that the swaps used are in the money and a payoff is expected. At this moment, the marked to market (MtM) value of the swap portfolio is positive and the contracts can be terminated against this value. To hedge the counterparty risk, there are 2 options: The first one is to buy credit default swaps on the counterparty, but these are complex products which do not always work as expected, such as during the 'voluntary writeoff' of Greek government bonds.[27]

The more common way to take away counterparty risk is to use collateral. This means that every day, the present value of the contract is posted as collateral on a separate account which can be taken if the counterparty defaults. ISDA(2011) finds that in 2010, 70% of all OTC derivatives transactions were subject to a collateral agreement, of which 81% was collateralized with cash, and most of the remainder with government securities. He (2000) states about this: 'The current industry practice has essentially removed (in a significant way if not completely) the risk of default by either counterparty so that, for all practical purposes, swaps shall be valued without the consideration of counterparty risk.' In this statement one can derive that the valuation of derivatives is influenced by using collateral, because the credit risk is taken away. The Euribor curve has an AA rating and is thus too high, leaving the daily EONIA rate as the discount rate for the value of the derivative. This is left out of the calculations, as it severely complicates valuation and hedging while the difference in value is too small to influence the general result. In practice, collateral management comes at a cost which will be part of the administrative fees.

There is a rather large side effect, however. Posting of collateral works two ways, meaning that the provider has to post collateral to the counterparty when the MtM value of the positions is negative. This happens when interest rates go up or when inflation goes down, so that the negative value of the swap portfolio is compensated by a lower present value of the RDA. This violates the assumption of earning the one year interest rate on the deposit, because part of the money has to be used to

[27] Credit default swaps pay out in the case of a credit event, with the ISDA deciding when that occurs. This leaves room to debate and thus uncertainty, as was the case with Greek government debt: http://www.bloomberg.com/news/2011-10-31/isda-says-greek-bond-deal-may-not-constitute-cds-credit-event.html

post collateral on which the *daily* interest rate is the return. This rate is usually lower but the spread varies significantly over time. The spread can be added to the cost of collateral management to obtain the total costs of taking away counterparty credit risk. There is a caveat here, because the swap positions will be closed if the collateral is not posted. If the collateral amount exceeds the total portfolio, problems might arise. A famous example of this kind of liquidity constraint is the case of Metallgeselschaft, which lost over 1.3 billion dollars when it had to close its futures positions because it was unable to post the collateral[28], even though it was certain that in the long run they would make a profit on the hedge (Hull, 2007).

To prevent such a disaster, the collateral that has to be posted in the simulations is registered and denoted as a percentage of total portfolio value. The descriptive statistics of this simulation are shown in table 7:

Table 7: Collateral limits descriptive statistics

t	15 years	30 years
Average	0.46%	0.32%
Volatility	15.93%	8.61%
Skew	0.68	0.38
Kurtosis	1.12	0.38
5th percentile	-22.86%	-13.04%
25th percentile	-10.49%	-5.59%
Median	-1.13%	-0.23%
75th percentile	9.68%	5.62%
95th percentile	28.62%	15.44%

The average is close to zero, which makes sense because when the provider receives collateral the amount is positive while paying collateral it is denoted with a minus sign. The volatility over the scenarios is lower after 30 years than after 15 years, because the duration of the RDA is lower after 30 years so that the sensitivity to the market variables is lower, and thus the swap notionals. The percentiles are interesting because they show that it is unlikely that the provider ever has to post more than the client invested. The most important measure is actually not mentioned in the table; the minimum value in all observations at any time during the 30 years is -51%, which means that that percentage of the portfolio had to be posted as collateral on the swap positions. If collateral is taken into account to the valuation, this number will be somewhat lower but it is safe to say that the probability of a collateral call that the provider will be unable to make is very low if not zero. One could even argue that 60 percent of the portfolio can be put into a one year or longer deposit anyway to earn the higher interest rate and save the residual for collateral calls. In the unlikely case that the collateral amount is more than 40 percent of the portfolio, the deposit can usually be terminated by paying a fine. Determination of the optimal ratio is left for further research however. The results here will be quantitatively affected by ignoring collateral but will not change the general conclusion.

[28]In futures contracts one speaks of margin calls instead of collateral, but the goal and effect is similar.

4.3.4. Product perspective

This section will treat issues concerning practical implementation, communication and costs.

An often cited reason for taking risk in pension investment is that a good pension becomes unaffordable when it is fully guaranteed. This might be true, but it does not mean that taking risk and having a guarantee are mutually exclusive. The RDA replicating strategy is the closest one can get to a risk free asset class in the investment portfolio. One can thus choose the level of certainty at the cost of expected return, which is a matter of preference. This preference is hard to establish, but in the context of a sector wide pension fund there is probably a minimum level of certainty where either participants or social partners can agree on. Another option is a law which states a minimum percentage in the risk free asset based on income, for example. The residual portfolio can then be allocated to risky assets attempting to obtain the risk premium. A good example of a solution of this sort is the sector pension fund for the flower and plant wholesale industry in the Netherlands, where a dynamic asset allocation with a minimum level of 40% in a portfolio that is more or less equal to the RDA replicating strategy used in this thesis. (de Jong et al., 2008) In the individual space, a guarantee can be offered to people that do not have access to a DB scheme, which includes the growing group of the self employed.

Given current technological standards, a client would like to be able see his account balance on the internet at every moment in time. There are communication issues here, because the absolute amount of money can vary widely over time while the funding ratio is not affected. The client could be scared when she sees that she lost 30% of the portfolio, but if the RDA became 30% cheaper this is not a problem. This is a case where in the information provision has to be framed in a specific way to make the client understand. Optimal framing and communication regarding these products is left for further research even though it is probably simpler than current systems. The product aims at preserving purchasing power, so the communication might stand with only the current nominal purchasing power, as this is an accurate reflection of the future real value.

The costs of all this boils down to the following:

1. Administrative costs
2. Trading costs
3. Collateral management costs
4. Profit for the provider

The exact amounts will depend on the competition in the industry and the innovativeness of the pension and insurance sector and financial markets. The latter has shown some financial engineering by introducing inflation swap trackers which allow investors to expose themselves to any side of HICP, US and UK inflation swaps over the full range of tenors. Deutsche Bank[29] and Barclays offer these products, which can help in simplifying the hedge and bringing costs down. Bodie and Prast (2007) state that individual DB products need not be more expensive than current pension products offering a similar guarantee.

[29] http://index.db.com. Bloomberg ticker: DBDSEI + tenor

5. Conclusion

Across the globe, risks and responsibilities are transferred from non-human entities to ordinary people who are taking a proper pension for granted, putting welfare in danger. General financial education is not capable of solving this problem and thus an industry that is accused of being unimaginative has to come up with a solution during a shift towards a new regulatory regime and a credit crisis. This is a complex task, to put it mildly. This paper makes an attempt to open up the way for the pension and insurance industry to provide consumers what they actually need by demonstrating that it is conceptually achievable to guarantee a real pension. Using the possibilities that new technologies in computer power and financial markets offer, the welfare maximizing product of a real deferred annuity is in sight. On the continuum of DC to DB, this paper supports the growing body of evidence that both bounds have great shortcomings and that the optimal solution lies somewhere in the middle. By combining a DC accumulation phase with DB benefits, the best of both worlds can be put to work. This implies a large cost-efficient collective that can benefit from professional knowledge and technology in order to provide a guaranteed indexed pension with clear ownership rights. This way everybody should get access to a minimum living standard during retirement, without closing the door to taking more risk when wished for.

In the proposed solution, interest rate and inflation derivatives protect the purchasing power of the DC investment over time, making conversion risk a non-factor. The resulting counterparty credit risk is taken away using collateral. The portfolio can be moved to another provider without much of a problem and is clearly defined. If the client wants to obtain the risk premium, this can be done by allocating more wealth away from this risk free portfolio towards a risky asset. Ideally, regulation is such that there is a minimum percentage allocated to the risk free asset at all times, with strong defaults protecting the majority of the consumers that are not interested in changing their allocation.

It will take an incredible amount of time and effort to get to this pension Walhalla, but the practical implementation issues described in this thesis should be solvable by industry experts and traders. In the shadow of an aging and individualizing society, the industry has to pick up this issue. Better sooner than later.

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7. Appendix

In order to replicate the theoretical value of a RDA, a number of tools are required. As discussed in the introduction, the interest and inflation rates are the two variables which determine the present value of a deferred annuity. In order to determine the value, a term structure stretching over the full horizon of the required product is required for both variables. These term structures are extracted from traded market instruments. The valuation of these instruments and the subsequent term structure building is discussed first in this chapter. We start with the valuation of the most important instrument, the interest rate swap.

7.1. Valuation of interest rate swaps

We will use the traditional approach to interest rate swap valuation, which treats a swap as a portfolio of forward contracts on the underlying floating interest rate. It is then valued by separately assessing both legs of the swap. The fixed leg is equal to a fixed rate bond without the notional being exchanged. The floating leg is equal to a floating rate bond without exchanged notional. At initiation, the value of both the legs is equal, making an interest rate swap a product which does not require an initial investment. The fixed rate which makes the value of the contract zero; the par swap rate; is quoted in the market.

Both legs are valued by discounting all the future cash flows. Since the future payments of the floating leg are unknown, forward rates are used for valuation. If this would not be accurate, arbitrage opportunities would occur.

For the sake of illustration, we value a simple 1-year swap with a notional of 1, with the floating leg paying the 6 month Euribor rate every 6 months and the fixed leg paying the agreed fixed rate annually. This is the common market practice in the Euro zone.[30] We begin by introducing the notation:

$N = Notional$

$\alpha_{t,k} = Daycount\ fraction\ from\ time\ t\ to\ time\ k$

$$The\ day\ count\ convention\ (DCC) of\ the\ fixed\ leg\ is\ \frac{30}{360}, while\ the\ floating\ leg\ has\ \frac{Actual}{360}$$

$$Alpha\ is\ the\ year\ fraction\ between\ 2\ dates\ counted\ according\ to\ the\ appropriate\ DCC.$$

$\mathcal{Z}_k = Euribor\ zero\ rate\ with\ maturity\ k$

$K_k = Euribor\ swap\ rate\ with\ maturity\ k$

$$D_{t,k} = Euribor\ discount\ factor\ from\ time\ t\ to\ time\ k\ ^{31} = \frac{1}{1 + \mathcal{Z}_k * \alpha_{t,k}} \qquad (1)$$

[30] For USD and GBP swaps, the leg tenors are different.

[31] Simple compounding is used here, since the underlying Euribor curve is simple compounded. Ametrano and Bianchetti (2009) also use simple compounding in their work.

$$F_{t,k} = Euribor\ forward\ zero\ rate\ from\ time\ t\ to\ time\ k^{32} = \frac{1}{\alpha_{t,k}}\left(\frac{D_{0,t}}{D_{0,k}} - 1\right) \tag{2}$$

We will now value a one year swap, which has two legs, each with a stream of cash flows that have to be discounted. The fixed leg pays annually and thus pays one fixed cash flow, which is valued as:

$$PV(fixed) = D_{0,1y} * \alpha_{0,1y} * K_{1y}$$

To value the floating leg, we first need to define the forward rates as function of discount factors.
In the simple compounded world, there would be arbitrage opportunities if the following relation would not hold for the two year case:

$$\left(1 + \mathcal{Z}_{1y} * \alpha_{0,1y}\right) * \left(1 + F_{1y,2y} * \alpha_{1y,2y}\right) = 1 + \mathcal{Z}_{2y} * \alpha_{0,2y}$$

The one to two year forward rate is then easily found as:

$$F_{1y,2y} = \left(\frac{1 + \mathcal{Z}_{2y} * \alpha_{0,2y}}{1 + \mathcal{Z}_{1y} * \alpha_{0,1y}} - 1\right) * \frac{1}{\alpha_{1y,2y}}$$

Then, we invert (1) to express discount factors as a function of zero rates:

$$\mathcal{Z}_k = \left(\frac{1}{D_{0,k}} - 1\right) * \frac{1}{\alpha_{0,k}}$$

We can use this result to rewrite the one to two year forward rate:

$$F_{1y,2y} = \left(\frac{1 + \left(\frac{1}{D_{0,2y}} - 1\right) * \frac{1}{\alpha_{0,2y}} * \alpha_{0,2y}}{1 + \left(\frac{1}{D_{0,1y}} - 1\right) * \frac{1}{\alpha_{0,1y}} * \alpha_{0,1y}} - 1\right) * \frac{1}{\alpha_{1y,2y}}$$

$$F_{1y,2y} = \frac{\frac{1}{D_{0,2y}}}{\frac{1}{D_{0,1y}}} * \frac{1}{\alpha_{1y,2y}} = \left(\frac{D_{0,1y}}{D_{0,2y}} - 1\right) * \frac{1}{\alpha_{1y,2y}}$$

This result is generalized in (2), and is required for valuing the floating leg of the one year interest rate swap. The floating leg is valued by discounting all cash flows; the payment in 6 months and the final payment in one year:

$$PV_0(flt) = \alpha_{0,6m} * F_{0,6m} * D_{0,6m} + \alpha_{6m,1y} * F_{6m,1y} * D_{0,1y}$$

$$By\ rewriting\ the\ forward\ rates\ using\ (2)\ we\ get:$$

$$PV_0(flt) = \alpha_{0,6m} * \frac{1}{\alpha_{0,6m}}\left(\frac{D_{0,0}}{D_{0,6m}} - 1\right) * D_{0,6m} + \alpha_{6m,1y} * \frac{1}{\alpha_{6m,1y}}\left(\frac{D_{0,6m}}{D_{0,1y}} - 1\right) * D_{0,1y}$$

$$PV_0(flt) = \left(\frac{D_{0,0}}{D_{0,6m}} - 1\right) * D_{0,6m} + \left(\frac{D_{0,6m}}{D_{0,1y}} - 1\right) * D_{0,1y}$$

[32] Derivation on the next page.

$$PV_0(flt) = D_{0,0} - D_{0,6m} + D_{0,6m} - D_{0,1y}$$

$$PV_0(flt) = 1 - D_{0,1y}$$

This result also holds for multi-year swaps; which makes it an easy pricing method.

The results can be generalized to the following[33]. Note that the alphas are differentiated here, because the floating leg has semi-annual periods while the fixed leg has annual periods.[34]

$$PV_t(fixed) = N * K_n * \sum_{k=1}^{n} D_{t,k} * \alpha_{k-1,k}^{fxd} \tag{3}$$

$$PV_t(floating) = N * \sum_{k=1}^{n} D_{t,k} * \alpha_{k-1,k}^{flt} * F_{k-1,k} \tag{4}$$

$$Which\ reduces\ to:$$

$$PV_t(floating) = N * \left(1 - D_{t,n}\right) \tag{5}$$

The value of a receiver swap is then the fixed leg minus the floating leg, while a payer swap is the opposite.

7.2. Curve construction

With the knowledge from the valuation section, market swaps can be analyzed. In particular, the fact that the value is zero at inception offers possibilities in combination with a market quote. For valuation of a large set of cash flows at varying maturities (such as a RDA), a zero curve is required so that the discount factor for each cash flow can be derived. The swap market is the most liquid interest rate market making swaps the best suited instruments.

The shortest instrument used to build the Euribor interest rate curve in this thesis is the one year cash instrument, while liquid instruments of for example one week and three months are available. However, since the short end lacks importance in this thesis due to the long horizons, the short end of the curve is not further refined. All other instruments used are plain vanilla interest rate swaps, up to 50 years maturity. This is shown in table 8.

[33] The fixed and floating leg are valued separately and not summed to obtain the value of a swap. The 'value of a swap' is trivial because it is not clear whether a payer or receiver swap is meant. If there is reference to the value of a swap in this thesis, the value to the counterparty being discussed is meant.
[34] From now on, every time it could be unclear what period is meant, the superscripts will be added.

Table 8: Bloomberg codes of data used

Maturity	Euribor	BEI
1y	EUSA1 index	EUSWI1 index
2y	EUSA2 curncy	EUSWI2 index
3y	EUSA3 curncy	EUSWI3 index
4y	EUSA4 curncy	EUSWI4 index
5y	EUSA5 curncy	EUSWI5 index
6y	EUSA6 curncy	EUSWI6 index
7y	EUSA7 curncy	EUSWI7 index
8y	EUSA8 curncy	EUSWI8 index
9y	EUSA9 curncy	EUSWI9 index
10y	EUSA10 curncy	EUSWI10 index
15y	EUSA15 curncy	EUSWI15 index
20y	EUSA20 curncy	EUSWI20 index
25y	EUSA25 curncy	EUSWI25 index
30y	EUSA30 curncy	EUSWI30 index
35y	EUSA35 curncy	EUSWI35 index
40y	EUSA40 curncy	EUSWI40 index
50y	EUSA50 curncy	EUSWI50 index

A zero curve is created from these instruments using a process generally referred to as bootstrapping. This is described next.

7.2.1. Curve building[35]

To extract the zero rate from a swap, the underlying discount factors have to be solved for. We can rewrite equations (3) and (5), given that the value of the floating leg should equal the value of the fixed leg:

$$K_n * \sum_{k=1}^{n} D_{t,k} * \alpha_{k-1,k}^{fxd} - \left(1 - D_{0,n}\right) = 0$$

Since the fixed rate is quoted in the market, the one year discount factor is the only unknown in the equation in the case of a one year swap; so that it can be solved for.

$$\left(D_{0,1y} * \alpha_{0,1y} * K_{1y}\right) - \left(1 - D_{0,1y}\right) = 0$$

$$D_{0,1y} * \left(\alpha_{0,1y} * K_{1y} + 1\right) = 1$$

$$D_{0,1y} = \frac{1}{\left(1 + K_{1y} * \alpha_{0,1y}\right)}$$

Note that this is equal to (1), as the one year swap rate equals the one year zero rate. The equation for the two year swap is then:

$$K_{2y} * \left(D_{0,1y} * \alpha_{0,1y} + D_{0,2y} * \alpha_{1y,2y}\right) - \left(1 - D_{0,2y}\right) = 0$$

[35] Subsection based on Hull (2007)

Since we have calculated $D_{0,1y}$ already, and K_{2y} is the swap rate quoted in the market for the 2-year instrument, we are left with only one unknown parameter; $D_{0,2y}$, which can be solved for:

$$K_{2y} * \left(D_{0,1y} * \alpha_{0,1y} + D_{0,2y} * \alpha_{1y,2y}\right) = 1 - D_{0,2y}$$

$$K_{2y} * D_{0,1y} * \alpha_{0,1y} + K_{2y} * D_{0,2y} * \alpha_{1y,2y} + D_{0,2y} = 1$$

$$D_{0,2y} * (1 + K_{2y} * \alpha_{1y,2y}) = 1 - K_{2y} * D_{0,1y} * \alpha_{0,1y}$$

$$D_{0,2y} = \frac{1 - K_{2y} * D_{0,1y} * \alpha_{0,1y}}{(1 + K_{2y} * \alpha_{1y,2y})}$$

This can be generalized for k year swaps up to maturity n to:

$$D_{0,n} = \frac{1 - K_n * (\sum_{k=1}^{n-1} D_{0,k} * \alpha_{k-1,k}^{fxd})}{(1 + K_n * \alpha_{k-1,k}^{fxd})} \tag{6}$$

This way, we can step-by-step solve for all the discount factors of the curve's instruments. Using (1) we can then convert the discount factors to zero rates to obtain the required curve.

As is visible in table 8, there are no (liquid) instruments for all maturities. For example, the 11 to 14 year Euribor swap quotes are missing. We then jump to the 15 year quote, but this means that there are five unknowns and only one equation. However, by linearly interpolating between the discount factors, the 11 to 14 year discount factors can be written as functions of the 10 and 15 year discount factors, so that only $D_{0,15y}$ is left unknown which can be solved for using (6).[36]

Once a curve is built, any cash flow at any future time can be properly discounted. For cash flows with a maturity of more than 50 years, the 50 year zero rate is assumed, implying flat extrapolation of the zero rates. This is done due to lack of better alternatives, and the observation that very long term rates correlate strongly.

These interpolation methods are also used when building the inflation term structure. These are needed to inflate the nominal payments of the RDA to real benefits. The instruments used for constructing this curve are zero coupon inflation swaps, which have the attractive feature that the market quotes can be directly used to build the inflation term structure. This is due to the zero-coupon characteristic, which is contrary to the interest rate swaps which pay out every period. Inflation swaps that use the yearly inflation are also available but this market is less liquid than the zero coupon market. The curve construction is then straightforward, as the break even inflation (BEI) market quotes can serve as grid points of the curve. The valuation of zero coupon inflation swaps is discussed next.

[36] Linear interpolation is not the best practice for curve construction, as described by Hagan and West (2006) However, a more complex method would not change the results here but would lead to complexity and slower simulations due to curve fitting, which is why the simplest method is applied.

7.3. Valuation of zero coupon inflation swaps[37]

A zero coupon inflation (ILZC) swap pays out the netted value of the two legs at maturity without any intermediate payments. Analogous to interest rate swaps, a receiver ILZC swap is worth the fixed leg minus the floating leg, with the payer swap being the exact opposite by definition.

The value of the fixed leg is the compounded break even inflation (BEI) rate quoted in the market:

$$PV_s(fixed) = N * D_{s,e} * ((1 + BEI_{e-s})^{e-s} - 1) \tag{7}$$
$$where\ s\ denotes\ the\ startdate\ and\ e\ the\ enddate\ of\ the\ contract$$

Note that the fixed rate is compounded here, contrary to the interest rate swaps which pay out every period.

The floating leg pays out the realized inflation over a certain period, which is not exactly the period during which the contract is active. This is due to the fact that realized inflation in a particular month is only published some time after this month, because the data has to be collected and processed by the statistical agencies responsible for this task. The delay is typically two to three months, causing a mismatch which is not present in the case of interest rate swaps. The floating leg of a five year inflation swap starting the first of January 2011 will typically pay out the realized inflation over the period 1st of October 2010 to 1st of October 2015.

Generalized, the value of the floating leg is:

$$PV_s(flt) = N * \ D_{s,e} * \left(\frac{I_{s-lag}}{I_{e-lag}} - 1\right) \tag{8}$$

$$Where\ I_{s-lag}\ is\ the\ starting\ value\ of\ the\ HICP\ index\ and\ I_{e-lag}\ is\ the\ value\ at\ the\ enddate\ specified\ in\ the\ contract$$

The final value of the index is unknown today, so that the forward value is used analogous to the interest rate swap case. This value can simply be derived from the BEI rates observed in the market:

$$F_s(I_e) = I_s * (1 + BEI_{e-s})^{e-s} \tag{9}$$

The forward index value is obtained by inflating the current index value with the break even inflation for the period of interest. The BEI quotes are such that the value of the inflation swap is zero at inception, similar to the interest rate swap market. If all the BEI forwards materialize, no payment will take place at maturity because both legs have equal value.

The procedure is slightly more complicated when we want to value an off-market inflation swap. Kerkhof (2005) describes seasonality effects that influence the value of the swap and have to be accounted for. However, we avoid this problem by using periods of one year instead of months in the simulations later in this thesis, making seasonality become irrelevant. It is still necessary to value off-market swaps though, which is done by slightly altering (8). If the inflation swap started one year ago, the index value is known for the first year of the contract. If we insert this value in (9) and inflate it with the appropriate BEI rate, the forward value for the end date can be calculated. We then get:

[37] Based on Kerkhof (2005).

$$F_{s+n}(I_e) = I_{s+n} * (1 + BEI_{e-s-n})^{e-s-n} \quad for\, n < (e-s) \tag{10}$$

So that the forward index value can be calculated at any point in time, which consequently can be used for marking to market the floating leg:

$$PV_n(flt) = N * \ D_{s+n,e} * \left(\frac{I_s}{F_{s+n}(I_e)} - 1\right) \tag{11}$$

The fixed leg retains the same nominal payment but is discounted for a shorter period:

$$PV_s(fixed) = N * D_{s+n,e} * ((1 + BEI_{contract})^{e-s} - 1) \tag{12}$$

Using (11) and (12) an inflation swap can be valued at any point in time, which is done during the scenario analysis.

7.4. Bucket distribution

Table 9: Bucket distribution (5 buckets)

Bucket	RDA Nominal deltas	Bucket Instrument	RDA BEI deltas	Bucket Instrument
1	$\Delta_1^N + \Delta_2^N + \Delta_3^N + \Delta_4^N + \Delta_5^N + \Delta_6^N + \Delta_7^N + \Delta_8^N + \Delta_9^N + \Delta_{10}^N$	K_{10y}	$\Delta_1^{BEI} + \Delta_2^{BEI} + \Delta_3^{BEI} + \Delta_4^{BEI} + \Delta_5^{BEI} + \Delta_6^{BEI} + \Delta_7^{BEI} + \Delta_8^{BEI} + \Delta_9^{BEI} + \Delta_{10}^{BEI}$	BEI_{10y}
2	$\Delta_{15}^N + \Delta_{20}^N$	K_{20y}	$\Delta_{15}^{BEI} + \Delta_{20}^{BEI}$	BEI_{20y}
3	$\Delta_{25}^N + \Delta_{30}^N$	K_{30y}	$\Delta_{25}^{BEI} + \Delta_{30}^{BEI}$	BEI_{30y}
4	$\Delta_{35}^N + \Delta_{40}^N$	K_{40y}	$\Delta_{35}^{BEI} + \Delta_{40}^{BEI}$	BEI_{40y}
5	$\Delta_{45}^N + \Delta_{50}^N$	K_{50y}	$\Delta_{45}^{BEI} + \Delta_{50}^{BEI}$	BEI_{50y}

Table 10: Bucket distribution (3 buckets)

Bucket	RDA Nominal deltas	Bucket Instrument	RDA BEI deltas	Bucket Instrument
1	$\Delta_1^N + \Delta_2^N + \Delta_3^N + \Delta_4^N + \Delta_5^N + \Delta_6^N + \Delta_7^N + \Delta_8^N + \Delta_9^N + \Delta_{10}^N$	K_{10y}	$\Delta_1^{BEI} + \Delta_2^{BEI} + \Delta_3^{BEI} + \Delta_4^{BEI} + \Delta_5^{BEI} + \Delta_6^{BEI} + \Delta_7^{BEI} + \Delta_8^{BEI} + \Delta_9^{BEI} + \Delta_{10}^{BEI}$	BEI_{10y}
2	$\Delta_{15}^N + \Delta_{20}^N + \Delta_{25}^N + \Delta_{30}^N$	K_{30y}	$\Delta_{15}^{BEI} + \Delta_{20}^{BEI} + \Delta_{25}^{BEI} + \Delta_{30}^{BEI}$	BEI_{30y}
3	$\Delta_{35}^N + \Delta_{40}^N + \Delta_{45}^N + \Delta_{50}^N$	K_{50y}	$\Delta_{35}^{BEI} + \Delta_{40}^{BEI} + \Delta_{45}^{BEI} + \Delta_{50}^{BEI}$	BEI_{50y}