A Framework for Examining Asset Allocation Alpha[1]

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Abstract

Despite the large literature on the importance of asset allocation as a primary determinant of portfolio performance, the definition of asset allocation "alpha" remains a poorly defined concept. In this paper, we show that a portfolio's total alpha can be decomposed into alpha from asset allocation and manager selection. The asset allocation alpha can then be further attributed to value-add from (1) taking additional risk exposure relative to the policy portfolio, (2) exploiting the relative value differential between assets with similar risk exposures and (3) timing the cyclicality in risk premia.

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Introduction

Manager alpha, in the finance practitioner parlance, is most often defined by an active portfolio's excess return over its benchmark.[4] In most situations, both the definition and the interpretation of manager alpha are straight-forward and non-controversial. However, asset allocation alpha remains a poorly defined concept. This is somewhat surprising given the large literature on the importance of asset allocation as a primary determinant of portfolio performance [5]. First and foremost, a theoretical framework and a well-accepted convention for defining a "passive" asset allocation benchmark portfolio are both conspicuously absent.

In both equity and fixed income investment management, a passive portfolio is one that mimics the "market" exposure for the desired market or segment. By this definition, the passive benchmark is a capitalization-weighted index, which reflects both the market-clearing portfolio and the performance of the average portfolio. In this construct, manager alpha is simply the excess performance of an active portfolio over the passive benchmark portfolio. More advanced factor analytics could further decompose the manager alpha into skill-based and risk-based components. For example, an active manager in the U.S. large cap space might be benchmarked to the Russell 1000® index. His excess return over the Russell 1000 would be considered "alpha" in most performance analyses. More sophisticated investors might further break that "alpha" into skill-based alpha, resulting from the manager's market timing or stock picking ability, and risk-based alpha, resulting from exposure to factors like value characteristics or illiquidity.

In practice, this framework is adequate and appropriate for all investors interested in investing in large cap U.S. active managed portfolios. However, it does not apply naturally to asset allocation investments. Defining a "default" passive asset allocation portfolio that would be universally appropriate for different investors is an exceptionally difficult if not hopeless task[6]. Certainly, a cap-weighted portfolio of all asset classes would not serve most investors' purposes. Canadian investors, for instance, would find it unacceptable for their asset allocation portfolios to track a benchmark that has negligible exposure to Canadian stocks and bonds due to the facts that the Canadian stock market is one-

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[4] In this paper, we use the more common industry definition of manager alpha. We do not use the academic definition of alpha, where the excess return over benchmark is further adjusted for any loading on known risk factors.

[5] See, for example Brinson, Hood, and Beebower (1986), Brinson, Singer and Beebower (1991), Ibbotson and Kaplan (2000), and Xiaong, Ibbotson, Idzorek, and Cheng (2010).

[6] See Doeswijk, Lam, and Swinkels (2012) for a recent effort towards computing a capitalization weighted global multi-asset portfolio for a broad range of investible asset classes.

tenth the size of the U.S. market and the Canadian government issues substantially less debt. Indeed, home country bias is a fact of life in investors' asset allocation decisions, for both rational and affective reasons. (The former might include regulatory constraints and investors' realistic appraisals of their own analytical limitations; the latter, their comfort levels and patriotic feelings.) These considerations rule out using the "market clearing" concept as the foundation for a generalized passive asset allocation benchmark.

Recognizing that different asset allocation portfolios are suitable for investors with different needs—for example, some seek to defease long-dated liabilities and others seek to generate a perpetual stream of real income[7]—it is probably too ambitious to attempt to establish a unifying structure for determining benchmark asset allocation portfolios. Instead, we propose a framework for thinking about asset allocation alpha assuming that investors have suitably determined their asset allocation policy portfolio.

Total Portfolio Alpha = Asset Allocation Alpha + Manager Selection Alpha

Once an investor has settled on an appropriate policy portfolio, measuring the performance of the actively managed asset mix becomes fairly straightforward. Generally, the total portfolio alpha, defined in this context as the excess return of the implemented portfolio over the policy portfolio, can be decomposed into asset allocation alpha and manager selection alpha.[8,9] To tease out the asset allocation alpha, we must first define another portfolio—the tactical portfolio. The tactical portfolio has the identical asset class exposure as the total portfolio; the only difference is that it only invests in the indexes assigned as benchmarks to each of the permissible asset classes. One can think of the tactical portfolio as the total portfolio implemented with only passive indexes. The difference in return between the total portfolio and the tactical portfolio can be intuitively interpreted as the manager selection alpha. The total portfolio will have a positive return advantage over the tactical portfolio if the active manager selection has been successful. The difference in return between the tactical portfolio and the policy portfolio is then interpreted as the asset allocation alpha, which is the main focus of this paper.

[7] Different investors may have different governance and restrictions as well as legacy and political considerations, which necessitate different passive benchmarks even if they have similar investment needs.
[8] The evidence on the lack of predictability on manager alpha has meant that manager selection alphas have largely been zero (or even negative). A sizeable Middle Eastern sovereign wealth fund famously proclaimed that, while they believe that skilled managers exist, the fund's professionals are simply not smart enough to identify them *ex ante*.
[9] This is in the spirit of Ibbotson and Kaplan (2000), where the authors decompose a portfolio's total return into a *policy return* (the returns due to the policy portfolio benchmark) and an *active return* (the returns due to manager's actively to over- or underweighting asset classes and securities). Our focus here is just on the manager's alpha from to over- and underweighting asset classes.

Let's consider a naïve 60/40 S&P 500/BarCap Aggregate policy portfolio and an implemented portfolio which allocates 40% to active U.S. large cap managers, 20% to active international large cap managers, 30% to active core bond managers, and 10% to emerging market bond managers. The asset allocation alpha would be the difference between the return of the policy portfolio and the return of a tactical portfolio that is invested 40% in the benchmark S&P 500 index, 20% in the MSCI EM index, 30% in BarCap Agg index and 10% in BarCap High Yield Bond index. The manager selection alpha would be the weighted average return in excess of the actively managed portfolio's benchmark returns.

However, the quantified asset allocation alpha is not a particularly valuable piece of information unless we can discern the nature of the outperformance and determine, with some confidence, whether the "alpha" is likely to persist. Specifically, we are interested in determining whether the measured asset allocation alpha is skill-based, risk-based, or just noise.

A Risk-Based Framework for Analyzing Asset Allocation

The customary asset allocation framework is based on convenience and, perhaps, an implicit assumption that key asset classes match up well with the important risk exposures. In accordance with this traditional practice, the allocation process involves assigning weights to the various asset classes available to the investor (equities, bonds, commodities, real estate, etc.). Asset classes are proxied by their corresponding market indexes. Each major asset category is further divided into sectors—United States, international, and emerging markets for equities, and U.S. Treasuries, international sovereigns, and corporates for bonds. In this framework, assets are vehicles for "owning" risk exposures, so the "asset-based" approach is, essentially, an "investment product-based" approach.

The more modern analytical framework is risk-based and, accordingly, makes a bright-line distinction between investment vehicles and risk exposures. In asset allocation parlance, individual asset classes contain different risk exposures. A desired combination of risks can be achieved with different asset allocation portfolios. Ultimately, prices, costs, and investment constraints will dictate the preferred portfolio. A useful analogy compares risks to nutrients, assets to foods, and portfolios to meals.[10] Because nutrients come bundled in various foods—dairy, grains, meats, for example—people must combine foods to create a nourishing meal. Given that different meals provide comparable nutrition, personal taste and food prices tend to determine the preferred meal.

[10] The nutrient vs. food analogy is not original; it has been used previously by Professor John Cochrane at the University of Chicago and Professor Andrew Ang at Columbia University.

The Three Sources of Asset Allocation Alpha

Risk-Based Asset Allocation Alpha

There are many asset classes but significantly fewer risk drivers for asset returns. Asset classes largely respond to shocks to a few common macro risk drivers, such as GDP growth, credit availability (funding liquidity), inflation, central bank policy, and geopolitical stability.[11] Risk factors can be constructed to capture these macro return drivers and used to assess the asset allocation alpha associated with discretionary risk taking. For example, the S&P 500 return is often used to measure shocks to growth expectations; credit spread to measure liquidity shocks; long Treasury yields to measure inflation shocks; currency baskets to measure central bank policy shocks; and commodities to measure geopolitical stability shocks. These five macro factors are the components of a convenient model for assessing a U.S.-centric investor's portfolio risk exposures.

To illustrate this model, let us return to the naïve 60/40 S&P500/BarCap Agg policy portfolio and the 40/20/30/10 S&P 500/MSCI EM/BarCap Agg/BarCap High Yield tactical portfolio. To determine whether, relative to the policy portfolio, the tactical portfolio has excess exposure to macro risks, we first determine the risk factor exposures for each of the asset class benchmark indexes.

Table 1. Factor Loadings for Asset Classes and Asset Allocation %

	Growth Factor Loading (equity)	Liquidity Factor Loading (credit)	Wt. in Policy Portfolio	Wt. in Tactical Portfolio
S&P 500	1	0.1	60%	40%
MSCI EM	1.2	0.3		20%
BarCap Agg	0.1	0.1	40%	30%
BarCap HY	0.5	0.7		10%

Table 1 provides a numerical example of factor loadings on the growth and liquidity risk factor for the above mentioned four asset classes. Both the policy and tactical portfolios have 60% allocated to stocks, and 40% allocated to bonds. However, the tactical portfolio has higher exposure to both the growth and the liquidity risk factors. These increased risk exposures, shown in Table 2, arise from including Emerging Market Stocks (which have a higher growth risk exposure than S&P 500) and High Yield Bonds (which have a higher liquidity risk exposure than the BarCap Aggregate).

[11] See the classical works by Chen, Roll and Ross (1986), Fama and French (1989), and Schwert (1990) on macroeconomic risk factors driving asset returns. See Cochrane (2005) and Ang, Goetzmann and Schaefer (2011) for overviews of the recent literature and empirical evidence.

Table 2. Factor Loadings for Policy vs. Tactical Portfolio		
	Growth Factor Loading (equity)	**Liquidity Factor Loading (credit)**
Policy Portfolio	0.64	0.1
Tactical Portfolio	0.72	0.2

Persistent higher levels of risk exposure in the tactical portfolio are likely to deliver outperformance against the policy portfolio. However, this source of alpha may be undesirable: higher risk exposures may unintentionally and unacceptably conflict with the risk objective and intended risk characteristics of the policy portfolio.[12] While risk-based "alpha" might be the most theoretically reliable source for long-term outperformance, it typically also generates the most significant tracking error against the policy portfolio reflecting a potential significant alteration of the intended portfolio risk characteristics.

Relative Valuation Trades as Asset Allocation Alpha

As we discussed in the previous section, the policy portfolio will have certain macro risk factor exposures. However, one can generally mimic similar risk exposure through combinations of assets other than those found in the policy mix. Given that different combinations of assets can provide similar portfolio risk characteristics (different meals with similar nutrients), judiciously allocating portfolio weights to the lowest priced combination would deliver the most favorable expected returns. The excess return, attributable to this skill, we refer to as the *relative valuation allocation alpha*.

Consider the following illustrative example of allocation alpha from relative valuation trades. In relative valuation trades, the portfolio manager does not express a view on the expected return of a risk factor; instead, he expresses a view on the relative attractiveness of the assets. For this example, we focus only on the growth risk factor exposure, but, in the general case, it is important to closely match exposures to other factors as well. Table 3 shows a tactical portfolio which has 30% allocated to stocks (split evenly across U.S., International and EM equities), 10% allocated to REITS, and 60% allocated to bonds (split evenly between investment grade and high yield).

[12] For the purist, we are admittedly abusing the language of "alpha." We consider excess return driven by exposure to systematic risk factors as a source for "measured" manager alpha against his benchmark.

Table 3. Factor Loading for Asset Classes and Asset Allocation %			
	Growth Factor Loading	Policy Portfolio	Tactical Portfolio
S&P 500	1	60%	10%
MSCI EAFE	1.1		10%
MSCI EM	1.2		10%
REITS	1.3		10%
BarCap Agg	0.1	40%	30%
BarCap HY	0.5		30%

The tactical portfolio is constructed specifically to mimic the risk factor exposures of the policy portfolio. In this numerical example, we display only the growth factor loading for brevity in exposition. See Table 4.

Table 4. Factor Loadings for Policy vs. Tactical Portfolio	
	Growth Factor Loading
Policy Portfolio	0.64
Tactical Portfolio	0.64

Here, the tactical portfolio has the same growth risk factor exposure as the policy portfolio, but with a very different asset mix: the tactical portfolio has 30% allocated to stocks, 10% allocated to REITS, and 60% allocated to bonds, whereas the policy portfolio is 60%/40% stocks/bonds. If we consider only a factor-based model of returns, the two portfolios should have the same expected return, because they have the same risk factor exposures. However, suppose the valuations—for example dividend yields and Shiller PE ratios for stocks, and credit spreads on bonds—suggest that international developed and emerging market stocks, as well as high yield bonds, are cheaper than U.S. Stocks as vehicles for accessing the growth factor. Then the portfolio manager can achieve asset allocation alpha by maintaining identical risk factor exposure to the policy portfolio, while shifting allocations to the more attractive assets.

Since asset class valuation levels can change, the relative valuation exercise is dynamic and the manager must respond to constantly shifting prices. However, it is important to understand that in so doing, the manager is largely maintaining a stable risk exposure over time even if his asset mix is highly volatile. For relative valuation trades, the manager usually expresses views on asset class "attractiveness" rather than on the underlying risk factors. A manager who steadily displays skill in the relative valuation trade will generally have the lower and more consistent tracking error over time.

Factor-Timing Trades as Asset Allocation Alpha

Just as asset class expected returns can fluctuate over time, so can the premia for risk factors. There is strong empirical evidence that risk premia are mean-reverting over time, reflecting the cyclical nature of investors' risk aversion over a full business cycle.[13]

A long-horizon investor can capture asset allocation alpha by exploiting the time-varying nature of risk premia through tactically shifting allocations over the course of a business cycle, while maintaining an average risk exposure similar to that of the policy portfolio. For example, a manager can overweight pro-cyclical assets (capturing exposure to the growth risk factor) when the growth risk premium is especially high and underweight pro-cyclical assets when the growth risk premium are especially low[14]. This "market-timing" portfolio strategy is consistent with the literature on time-varying risk aversion, where at the tail end of most expansionary cycles, the market's risk aversion is muted due to the significant run-up in portfolio wealth and the over-extrapolation of recent growth rates. Flushed with cash and overly optimistic about the economy, investors are then too willing to speculate and to provide capital. In this environment the growth and credit factors are generally priced too richly, suggesting low forward returns to assets bearing heavy growth and credit exposures. We observe the opposite situation at the start of a recessionary phase, where risk aversion is high relative to the cyclical average.[15]

Table 5. Asset Allocation over a Business Cycle

	Growth Factor Loading	Wt. in Policy Portfolio	Tactical Portfolio Wt. During Recession	Tactical Portfolio Wt. During Expansion	Tactical Portfolio Wt. Average over Business Cycle
S&P 500	1	60%	80%	40%	60%
BarCap Agg	0.1	40%	20%	60%	40%

[13] See John Cochrane's 2011 American Finance Association Presidential Address for a review on the empirical literature documenting mean-reverting risk premium for a large variety of risk factors and assets.
[14] See Ang and Kjaer (2011) for a more detailed framework on how long-horizon investors can exploit time-varying risk premia through counter-cyclical investing.
[15] See Campbell and Cochrane (1999) for a rigorous model on time-varying risk aversion. See Hsu (2012) for a non-technical exposition of the same phenomenon.

Table 6. Factor Loadings (Portfolios)

	Growth Factor Loading
Policy Portfolio	0.64
Tactical Portfolio (Recession)	0.82
Tactical Portfolio (Expansion)	0.46
Tactical Portfolio (Average)	0.64

Table 5 shows a tactical portfolio that allocates counter-cyclically to the S&P 500 to capture add value from mean-reverting equity premium. Table 6 shows the resulting time-varying factor loading of the tactical portfolio. The tactical portfolio displays over-weight (relative to the policy portfolio) to the growth factor during recession and under-weight to the growth factor during expansion. Anecdotally, in 2007 all growth-oriented assets, such as equities and high yield bonds were trading near all-time high valuation levels after an extended period of strong growth which inflated asset prices and boosted optimism. As a result, the growth risk factor was not priced to deliver its usual risk premium; in fact, prices were arguably so high that the expected return to the factor had turned negative. In 2009, after the Global Financial Crisis triggered fire sales, the reverse occurred: growth assets were priced near historically low valuation levels, offering substantially above average expected returns. Our counter-cyclical tactical portfolio would have produced a substantial factor-timing alpha over the 2007–2010 period.

Managers who execute a factor-timing allocation strategy will be actively and tactically changing the portfolio's risk profile over time relative to the policy portfolio. The resultant tracking error will likely be dynamic and can be quite large if the magnitude of risk premium variation for factors is substantial.

A Numerical Example

In this section we provide a detailed numerical example on computing asset allocation alpha and decomposing the value-add into components due to risk factor exposures, relative valuation trades and factor-timing. For the exact formulas for the calculations, please see Appendix A.

Consider a two-period example with an asset class universe consisting of the following indexes: S&P500, MSCI Emerging Market Equities, BarCap Agg and BarCap High Yield. Table 7 shows the asset allocations of the policy portfolio, as well as the tactical allocations at the beginning of periods 1 and 2. Table 7 also shows the realized returns of the asset classes during periods 1 and 2.

Table 7. Asset Allocations and Realized Returns

Asset Class	Policy Portfolio Allocation (Period 1 & 2)	Tactical Portfolio Allocation (Average)	Tactical Portfolio Allocation (Period 1)	Tactical Portfolio Allocation (Period 2)	Realized Return (Average)	Realized Return (Period 1)	Realized Return (Period 2)
S&P 500	60%	40%	50%	30%	6.3%	10.5%	2.1%
MSCI EM		20%	30%	10%	8.4%	10.8%	5.9%
BarCap Agg	40%	30%	10%	50%	3.5%	3.1%	3.9%
BarCap HY		10%	10%	10%	6.9%	8.6%	5.2%

As is shown in the table, the portfolio manager has an average allocation that deviates from the policy portfolio (overweight EM equities and high yield bonds). The manager also both tactically shifts the portfolio weights through time (overweight equities in period 1 and overweight bonds in period 2). Based on the data in Table 7, we compute the returns of the policy and tactical portfolios as shown in Table 8. We see that the tactical portfolio had an average return of 6.7%, while the policy portfolio had an average return of 5.2%. Thus, the portfolio manager had an Asset Allocation Alpha of 1.5% over periods 1 and 2.

Table 8. Portfolio Realized Returns

Realized Portfolio Returns	Average Return	Return (Period 1)	Return (period 2)
Policy Portfolio	5.2%	7.5%	2.8%
Tactical Portfolio	6.7%	9.7%	3.7%

In the spirit of Hsu, Kalesnik and Myers (2010), we first decompose the asset allocation alpha into a static component (due to the average portfolio weight), and a dynamic component (due to time-varying portfolio weights). We then decompose the static allocation alpha into *risk-based* and *relative valuation* based alpha, and we decompose the dynamic allocation alpha into a *factor-timing* alpha and an unexplained residual term.

The static allocation alpha can be computed simply as the difference between the average tactical weights and policy portfolio weights times the average asset class returns. In this example, the static allocation alpha is ***0.75%***; that is, the average over-weights to EM equities and high yield bonds have added 75bps. The deviation in allocation from that of policy portfolio can be driven by the tactical portfolio (1) targeting a different mixture of risk factor exposures or (2) using cheaper assets to target a similar risk exposure as the policy portfolio. We examine the policy and tactical portfolios' risk-factor exposures to further decompose the static allocation alpha into risk-based and relative valuation-based alphas. In this example, we consider three risk factors: growth, duration and credit. Table 9 displays the risk factor loadings of the asset classes and those of the policy and tactical portfolios; note that the portfolio risk factor loadings can be easily computed using the asset class risk factor loadings and portfolio weights from Table 7.

Table 9. Factor Loadings for Asset Classes and Portfolios.

	S&P 500	MSCI EM	BarCap Agg	BarCap HY	Policy Portfolio	Tactical Portfolio (Average)	Tactical Portfolio (Period 1)	Tactical Portfolio (Period 2)
Growth Factor	1.00	1.20	0.10	0.50	0.64	0.72	0.92	0.52
Duration Factor	0.00	0.00	0.80	0.50	0.32	0.29	0.13	0.45
Credit Factor	0.10	0.30	0.10	0.70	0.10	0.20	0.22	0.18

We can see that, on average, the tactical portfolio has higher growth risk exposure (0.72) than that of the policy portfolio (0.64) and a higher credit risk exposure (0.20) than that of the policy portfolio (0.10). We also see that the portfolio manager is engaging in factor-timing, by taking higher than average growth and credit risk exposures in period 1 (a "risk on" tactical shift), and higher than average duration risk exposure in period 2 (a "risk off" tactical shift). The realized returns for the three risk-factors during for periods 1 and 2 are given Table 10. Note that in our numerical example, the pro-cyclical risk exposures (growth and credit) deliver stronger returns in period 1, while the counter-cyclical risk exposure (duration) delivers stronger returns in period 2. From a business cycle perspective, this would suggest that period 1 is more expansionary, while period 2 is more recessionary.

Table 10. Realized Factor Returns

Return	Average	Period 1	Period 2
Growth Factor	5.0%	7.0%	3.0%
Duration Factor	3.0%	2.0%	4.0%
Credit Factor	2.0%	3.0%	1.0%

To compute the risk-based alpha, we first need to compute the average factor loadings of the tactical and policy portfolios and then compute the average risk-based returns for the portfolios (by "risk-based return" we mean the component of return due to risk-factor exposures). The risk-based return of the tactical portfolio is computed using the average tactical portfolio factor loadings and the average realized factor returns. By subtracting the corresponding risk-based return of the policy portfolio, we find that the risk-based asset allocation alpha is ***0.51%***; that is the additional growth and credit risk exposure have added value. Subtracting the risk-based alpha from the total static allocation alpha of 0.75% reveals that ***0.24%*** of the asset allocation alpha came from relative valuation trades; that is the tactical portfolio successfully used a mixture of cheaper assets to create the desired risk exposure.

Finally we compute the allocation alpha from factor-timing. In our example, the *dynamic* allocation alpha is ***0.75%.*** The alpha from factor-timing can be computed as the difference between the average of the factor-based returns and the average of factor loadings times the average factor return. In our example, the portfolio manager produced 0.58% additional return through factor-timing. Since the dynamic allocation alpha was 0.75%, this leaves 0.17% as an unexplained residual. Table 11 summarizes all of the alpha computations in our numerical example.

Table 11. Decomposition of Asset Allocation Alpha

Asset Allocation Alpha	**1.50%**	*Value-Add of Tactical Portfolio over Policy Portfolio*
Static	0.75%	*(Avg Tactical Wt - Policy Wt) * (Avg Asset Returns)*
Risk-Based	**0.51%**	*(Avg Tactical β -Policy β) * (Avg Factor Returns)*
Relative Valuation	**0.24%**	*Static Alpha - Risk-based Alpha*
Dynamic	0.75%	*Avg (Tactical Wt * Asset Returns) - (Avg Tactical Wt)*(Avg Asset Returns)*
Factor-Timing	**0.58%**	*Avg (Tactical β * Factor Returns) - (Avg Tactical β)*(Avg Factor Returns)*
Unexplained Residual	**0.17%**	*Dynamic Alpha - Factor Timing*

Conclusion

The excess return of the investor's total portfolio over his policy benchmark can be decomposed into asset allocation alpha and manager selection alpha. The former can further be attributed to (1) taking additional risk exposure relative to the policy portfolio, (2) exploiting the relative value differential between assets with similar risk exposures and (3) timing the cyclicality in risk premia. When we carefully measure and attribute a manager's asset allocation alpha, we are then more able to understand the nature of the manager's skill and how his active deviations impact our portfolio risk objectives.

Appendix A: Mathematical Derivations

For asset i at time period t, define the tactical weight at the beginning of the period as $w_{i,t}$, the benchmark policy weight as $w_{b,i}$ and the realized return during the period as $R_{i,t}$. The asset allocation alpha is given by:

$$\text{Asset Allocation Alpha} = \frac{1}{T}\sum_{t}\sum_{i}\left(w_{i,t} - w_{b,i}\right)R_{i,t}$$

Denote the average tactical weight as $\bar{w}_i = \frac{1}{T}\Sigma_t\, w_{i,t}$ and the average asset return as $\bar{R}_i = \frac{1}{T}\Sigma_t\, R_{i,t}$. The alpha can be decomposed into *static* and *dynamic* components as follows:

$$\text{Static Allocation Alpha} = \sum_{i}\left(\bar{w}_i - w_{b,i}\right)\bar{R}_i$$

$$\text{Dynamic Allocation Alpha} = \frac{1}{T}\sum_{t}\sum_{i}\left(w_{i,t} - \bar{w}_i\right)R_{i,t}$$

Suppose asset returns are driven by multiple risk factors as in the APT framework of Ross (1976):

$$R_{i,t} = \alpha_i + \sum_{k}\beta_{i,k}F_{k,t} + \varepsilon_{i,t}$$

and denote the average risk factor returns as $\bar{F}_k = \frac{1}{T}\Sigma_t\, F_{k,t}$. Then we can decompose the static allocation alpha as into a *risk-based* and *relative valuation* alphas as follows:

$$\text{Risk} - \text{based Alpha} = \sum_{i}\sum_{k}\left(\bar{w}_i - w_{b,i}\right)\beta_{i,k}\,\bar{F}_k$$

$$\text{Relative Valuation Alpha} = \sum_{i}\left(\bar{w}_i - w_{b,i}\right)\alpha_i$$

Notice that in a portfolio manager's risk-based alpha comes from an increased exposure to factor risk premia. In contrast, relative valuation alpha comes from a manager's overweight to assets which are underpriced relative to their factor-based expected returns (assets with a high expected α_i).

Finally, we can decompose the dynamic allocation alpha into a component due to factor-timing and an unexplained residual component.

$$\text{Factor} - \text{Timing Alpha} = \frac{1}{T}\sum_{t}\sum_{i}\sum_{k}\left(w_{i,t} - \overline{w}_{i}\right)\beta_{i,k}\,F_{k,t}$$

$$\text{Unexplained Residual} = \frac{1}{T}\sum_{t}\sum_{i}\left(w_{i,t} - \overline{w}_{i}\right)\varepsilon_{i,t}$$

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Important Information