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QUANTITATIVE STRATEGIES:
FACTOR INVESTING

Total Portfolio Factor, Not Just Asset, Allocation

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Strategic portfolio allocation is built on the foundation of diversification. As more assets are added to a portfolio, the idiosyncratic (or asset-specific) risk can be diversified away, leaving exposures to a parsimonious set of factors—broad, persistent drivers of return. The portfolio's profile in factor risk terms is the key determinant of diversification benefits because it is factor risk that manifests itself in many different asset class returns and can carry long-term rewards. The focus of strategic portfolio allocation, however, has been largely through an asset lens. This is primarily a legacy of using assets as direct inputs into mean–variance optimization routines introduced by Markowitz [1952] and more recently Swensen [2000], dividing even diverse private market investments into discrete groups of asset classes.

We present a framework for strategic portfolio allocation in terms of macro factors. The framework entails the following:

1. measuring the current portfolio's factor exposures across all assets, including liquid and illiquid markets;
2. determining optimal factor, rather than asset class, exposures; and
3. mapping the desired factor exposures to the best mix of private and public markets, subject to investor preferences and tolerances over illiquidity and other constraints.

The framework serves as a practical implementation of the factor allocation process, which involves *ex ante* decisions about which factor premiums are appropriate followed by a determination of which assets can deliver those factor exposures. Our intention is to reframe asset allocation and portfolio analysis along factor dimensions and provide a comprehensive implementation of a factor-based strategic allocation workflow in a simple, transparent, and intuitive process. We apply the factor allocation framework to representative institutional portfolios and show how superior investment outcomes may result by using a factor analysis.

There are numerous benefits to adopting a factor-based allocation workflow to complement an asset-based approach. Indeed, 9 out of 10 large institutional investors are already using factor analysis in some part of their investment process to increase diversification, enhance risk–return trade-offs, and reduce costs.[1] During the financial crisis in 2008 and 2009, traditional asset allocation approaches failed to deliver effective diversification for many institutions; many asset classes moved in lockstep, an observation consistent with an underlying multifactor model in which assets are exposed to a common set of factors.

Factors can enhance asset allocation decisions by highlighting portfolio-level sensitivities to markets and events. A handful of factors have tended to dominate total

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and active risk—especially economic growth, real rates, and inflation—yet many institutions attempt to predict market behavior and estimate returns for dozens of asset classes. A parsimonious set of explanatory factors can reduce the complexity of such assumptions and allow institutions to focus research efforts on the return drivers that matter most. Factors can also be employed defensively to potentially achieve better risk–return trade-offs and reduce drawdown risk in times of uncertainty or market stress (Clarke, de Silva, and Murdock [2005]; Ang, Goetzmann, and Schaefer [2009]). Moreover, tradable factor-mimicking portfolios for macro factors can be constructed (see, e.g., Chen, Roll, and Ross [1986]), which offer a cost-effective means to acquire desired factor exposures. Low-cost factor benchmarks can be further used to identify true alpha in excess of factors and market indices (Kahn and Lemmon [2016]; Ang, Goetzmann, and Schaefer [2009]).

FACTOR VIEWS

We work with six macro factors and a broad, global range of asset classes; however, our examples are illustrative and our framework can be applied to any set of factors and asset classes.

Selecting a Parsimonious Set of Macro Factors

We select a set of macro factors motivated from principal components analysis of the covariance matrix of monthly returns from January 1997 to September 2015 for 13 global asset classes.[2] We use returns hedged to U.S. dollars. The first six principal components explain 95% of the comovement of these asset class returns, with the first three principal components accounting for 85% of the cross-asset movements.

It is possible to interpret the first few principal components as macro factors. For example, the first principal component correlates strongly with an equal-weighted portfolio of risky assets and the second correlates with a blend of safe haven assets; as such, we infer the presence of economic growth, real rates, and inflation factors. We can identify three additional macro factors—credit, emerging markets, and commodity—through further empirical analysis of the remaining principal components. Thus, following Greenberg, Babu, and Ang [2016], we define the set of macro factors in Exhibit 1: economic growth, real rates, inflation, credit, emerging markets, and commodity. These factors are economically intuitive, and Exhibit 1 also defines sensible, tradable representations of each of the macro factors with low-cost asset class indices. The choice of six macro factors is in line with the recommendations of Blyth, Szigety, and Xia [2016], and others, who found this an adequate number in practical applications to explain a large amount of the variation of asset returns, yet maintain a requisite small number for ease of constructing strategic portfolios allocations.[3]

By construction, the six macro factors do not capture currency risk, so we introduce an additional foreign currency (FX) factor. FX is not a rewarded macro factor in that it does not have a long-run return premium (see Campbell, Serfaty-de Medeiros, and Viceira [2010]), but FX is an important driver of portfolio volatility.

EXHIBIT 1
Macro Factors

Economic Growth	Risk associated with global economic growth *Broad-market equity index returns*
Real Rates	Risk of bearing exposure to real interest rate changes *Inflation-linked bond returns*
Inflation	Risk of bearing exposure to changes in nominal prices *Return of long nominal bonds, short inflation-linked bonds portfolio*
Credit	Risk of default or spread widening *Return of long corporate bonds, short nominal bonds portfolio*
Emerging Markets	Risk that emerging sovereign governments will change capital market rules *Basket of EM equity premia, EM CDX, and EM FX*
Commodity	Risk associated with commodity markets *Weighted GSCI Commodity index returns*

Mapping Assets to Macro Factors

We assume the following macro factor representation of each asset class return i:

$$r_i = a_i + \sum_j b_{ij} f_j + \varepsilon_i \tag{1}$$

for the macro factors, f_j, where $j = 1 \ldots 6$.

To compute the macro factor exposures, b_{ij}, in Equation 1, we work directly with risk characteristics from a risk model. (We reserve the term "factor" for the broad, persistent macro factors defined in Exhibit 1.) These risk characteristics include standard variables like the beta or book-to-price ratio for a stock, and other security-specific variables like the OAS spread of a certain structured product. Many risk models map assets or indices onto potentially hundreds, and often thousands, of risk characteristics.

The risk model gives a representation of each asset's return r_i into risk characteristics, γ_{ik}, with returns g_k, for $k = 1 \ldots K$, where the number of characteristics K is usually several orders of magnitude larger than the number of macro factors:

$$r_i = \alpha_i + \sum_k \gamma_{ik} g_k + u_i \tag{2}$$

We use the risk characteristics to estimate the macro factor loadings in Equation 1 by creating a sparse "translation matrix" between macro factors and characteristics that has the dimension of six factors by several thousand risk characteristics. Each asset class has a risk characteristic representation, and thus we can use the translation matrix to obtain its macro factor loadings.

We estimate the translation matrix as follows. We regress the risk characteristics, γ_{ik}, onto the macro factors such that:[4]

$$\gamma_{ik} = b^*_{i1} f_1 + \ldots + b^*_{i6} f_6 + b^*_{iFX} f_{FX} + \nu_i \tag{3}$$

The regressions in Equation 3 are constrained and informed by economic priors. For example, we impose the constraint that a government bond should have exposure to only the real rates and inflation factors, not economic growth. We also apply a set of step-wise regressions to keep the estimated coefficients small. More details of this mapping procedure, which follow Meucci [2007], are found in the Appendix.

Using this procedure, the factor approach reveals the common drivers behind all assets held—both public and private—in an asset owner's strategic policy portfolio. In Exhibit 2, we illustrate the macro factor contributions to risk for select asset classes commonly held by institutional investors. We observe that economic growth is the primary risk driver for global public equities, and real rates and inflation are the main contributors to global aggregate bond risk. Not surprisingly, a portfolio of bonds and equities brings together economic growth with real rates and inflation factors—which is the factor foundation of the traditional diversified stock–bond portfolio.

Exhibit 2 shows that the economic growth factor is responsible for more than 50% of the total risk of both global public equity and global private equity.[5] The factor decomposition reveals that some private asset classes are exposed to the same factor drivers as their public counterparts. The exposure to economic growth for private equity is larger because of greater leverage in this alternative asset class. The "Other" category—which represents the risk unexplained by the macro factors—is also larger, reflecting the larger specific, idiosyncratic component of private equity compared with public equity. It is from the idiosyncratic risk that private equity managers may generate alpha in excess of the macro factors (see Jensen [1968]). Global aggregate bonds are exposed to real rates, inflation, and credit factors—with the last factor having the smallest exposure. Global real estate prices have tended to increase when economic growth is high, giving rise to a positive exposure on the economic growth factor. The asset class also has characteristics similar to bonds in that it provides steady cash flows, which is behind the real rates and inflation factor exposures. Like private equity, there is a relatively large idiosyncratic component in real estate.

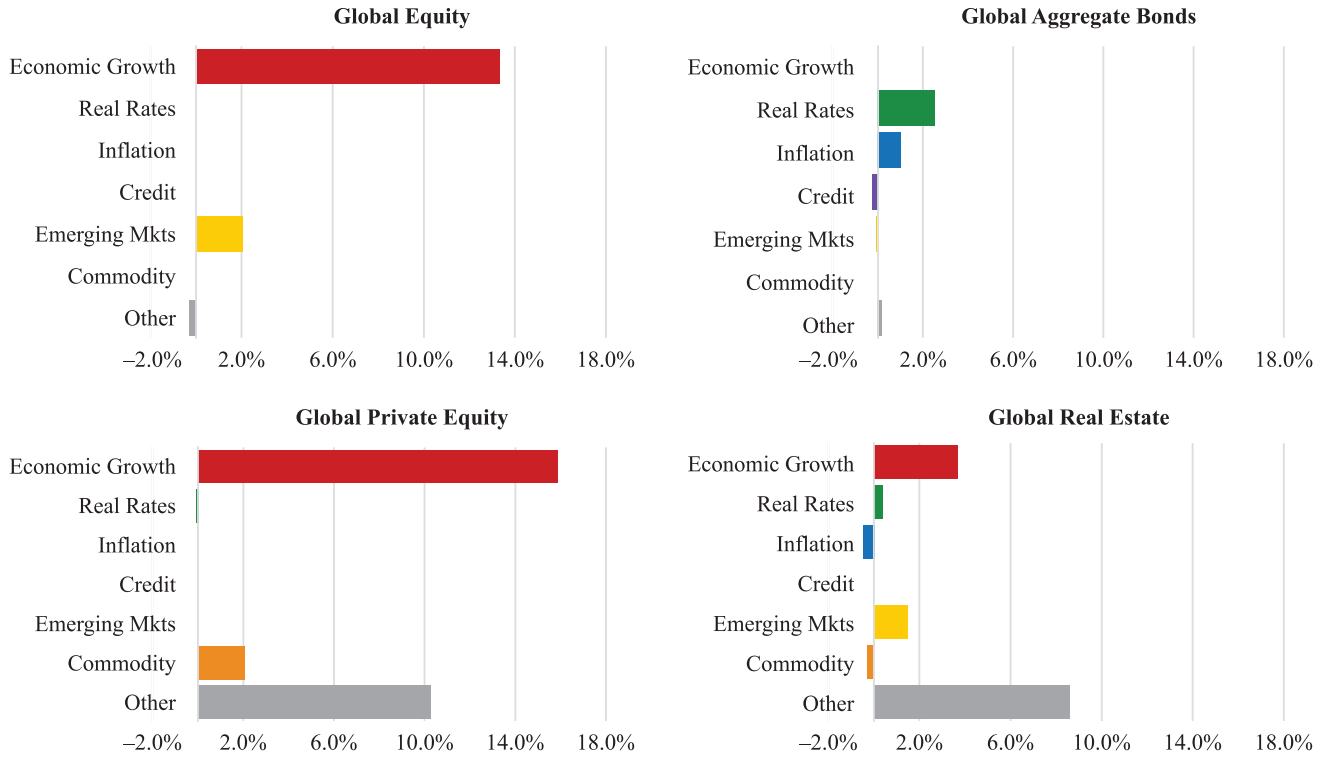
Factor Views of Institutional Portfolios

Exhibit 3 reports the macro factor risk profiles for three typical U.S. institutional portfolios: an endowment, a life insurer, and a public defined benefit plan.[6] Panel A reports the capital (dollar) allocations of the broad asset classes, and Panel B reports the risk contributions by macro factors.

For the endowment portfolio, economic growth is the primary driver of portfolio risk, contributing over two-thirds of the 13.1% annualized volatility due

EXHIBIT 2

Asset Class Risk Contributions by Macro Factors



Notes: Asset classes are mapped to representative market indices as follows: Global Equity to MSCI All Country World Index, Global Aggregate Bonds to Barclays Global Aggregate Index, and Global Private Equity and Global Real Estate to BlackRock proxies to represent asset classes where historical data is either lacking or of poor quality.

Source: Aladdin Factor Workbench, as of September 30, 2016.

to the substantial portfolio allocation to equity (36%) and alternatives (51%). This allocation aligns well with an endowment's comparative advantages: a long time horizon, large size, limited short-term liquidity needs, and a total return–based investment objective. Endowments can often bear the risk of outsized equity market or liquidity tail events more easily than the average investor; for bearing this risk, the endowment owner hopes to harvest significant excess returns.

The life insurer, by contrast, has a liability-based investment objective and more aggressive capital preservation and liquidity requirements. We observe that its portfolio is dominated by fixed-income assets (88%) with a 4.0% annualized volatility composed of real rates (64.3%), inflation (7.5%), and credit (11.0%) factor risks—in line with a liability-managed plan with conservative risk limitations.

The public defined benefit plan has similarities to both the total return–oriented endowment and liability-driven insurer. The typical pension plan has a mid to long time horizon, with increasing liquidity demands as liabilities become due, and its liabilities have high inflation, interest rate, and drawdown risk sensitivity. Some defined benefit plan sponsors have an investment objective that involves seeking a 100% funding ratio, which often requires managing a liability-hedging portfolio alongside a return-seeking portfolio that transitions to safe assets as the funding status improves; others use established plan discount rates and tilt toward total return–oriented objectives. Given these objectives and constraints, we expect pension plan owners to implement portfolios targeting a broad array of factor premia; yet, we see that 77% of the typical public pension plan's risk is driven by economic growth. A more

balanced macro factor approach with diversified exposures across factors and increased allocations to real rates and inflation may provide a superior risk–return trade-off and protect against downside market risk, as we now discuss.

OPTIMAL FACTOR ALLOCATIONS

The second part of the factor allocation framework involves determining the optimal factor, rather than asset class, exposures. There is no unique set of factors for all investors. Rather, institutions must set their desired factor allocation in the context of their investment objectives, liabilities, and constraints. Each factor defines a unique set of "bad times," or periods of underperformance (see Ang [2014]). As such, investors must select factor exposures that cater to their comparative advantages and compensate them adequately for the accepted factor risks.

Some commentators like Lee [2011] and Cocoma et al. [2016] argued that there is no additional benefit in analyzing a portfolio in factor terms, rather than traditional asset classes. It is true that under specific circumstances, the factor view is equivalent to an asset class view. We show these conditions in the Appendix and note they are rarely met in practice: The factors are formed only from the given set of assets; there are no constraints on the transformations required to map between factors and assets; and there are no alpha assumptions. The first condition does not hold in many settings: For example, there are many proxies of economic growth like GDP and industrial production that are not directly tradable. In practice, the second condition also does not hold as institutions often have leverage, shorting, and asset class holding requirements. Finally, many institutions invest in particular assets explicitly because they believe they can access alpha opportunities. Any violation of these conditions results in non-equivalence between factors and assets.

To illustrate the benefits of applying a factor lens to institutional portfolios, we modify the original portfolios in Exhibit 3 to more balanced, diversified portfolios of factors. We do not claim that this represents an optimal factor holding—our approach is deliberately simple and can be refined to construct an optimized portfolio. What is key is that we are able to obtain this diversification by working in terms of factors.

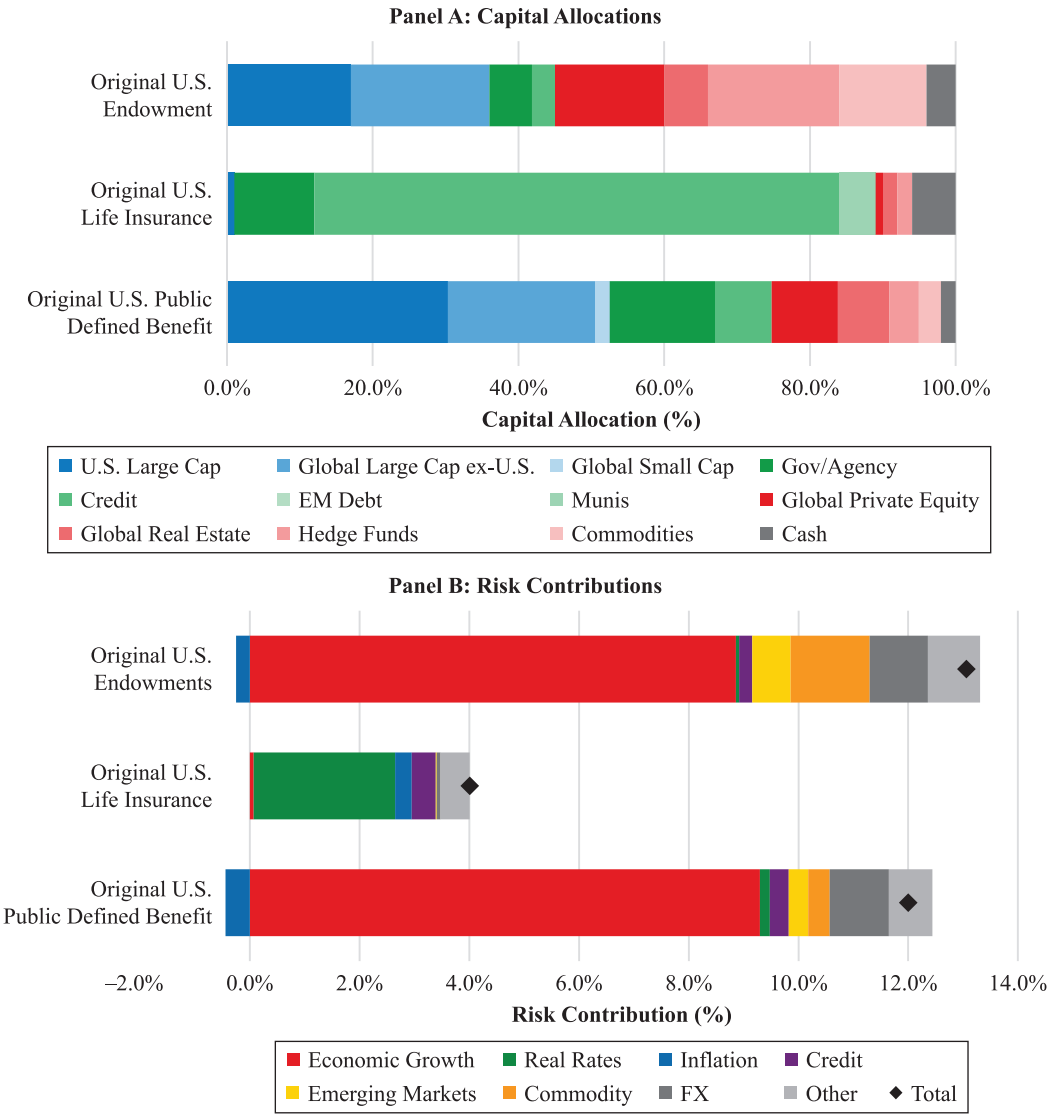
Risk–Return Improvements from More Balanced Macro Factor Exposures

We modify the representative institutional portfolios (endowment, life insurer, public pension plan) by overlaying the hypothetical factor completion portfolios in Exhibit 4, which presents each factor position as an overlay alongside a self-financed asset class implementation of the factor portfolio. We report factor exposures in the left-hand column and capital allocation weights in the right-hand column. We use factor completion overlay portfolios to move the factor risk profiles of the existing policy portfolios toward desired target portfolios of factors.

We now demonstrate that we can achieve superior risk–reward trade-offs for each portfolio in Exhibit 3 by explicitly diversifying across factors. We note that we do not seek to make investment policy recommendations; instead, we aim to emphasize the potential benefits of more balanced factor exposures through transparent and simple changes to the total portfolio:

1. For the U.S. endowment portfolio with substantial economic growth exposure, we overlay a portfolio that pares down economic growth exposure while broadly increasing exposure to other factors. The resulting modified portfolio increases the endowment manager's alpha budget by adding real assets, inflation-linked bonds, and emerging market debt to the portfolio while reducing the allocation to growth-oriented assets.
2. We seek to limit drawdown risk and enhance returns for the U.S. life insurance portfolio by introducing a global, diversified, market-neutral investment strategy to the portfolio. Such an approach introduces a new source of potential returns with limited impact to the portfolio's macro factor risk profile; thus, we make risk–return ratio improvements while satisfying the asset owner's risk constraints.
3. To seek to reduce portfolio risk while respecting long term expected return targets for the U.S. public defined benefit plan, we construct a factor overlay that adds relatively large allocations to non-growth related risk premiums. The modified policy portfolio diversifies the plan's return streams without changing existing private asset allocations.

EXHIBIT 3
Macro Factor Views of Institutional Portfolios



Notes: Hypothetical asset allocations for the representative institutions in Exhibits 3 through 9 are constructed according to the methodology in Note 6. Risk is defined as one standard deviation, annualized volatility. Factor exposures are defined as the portfolio betas to the respective factor returns. Capital allocation is denoted as percentage of NAV. The completion overlay portfolios presented in Exhibits 4 and 9 represent the changes required to move from the original portfolio to the modified portfolio in terms of both factor exposures and asset allocation.

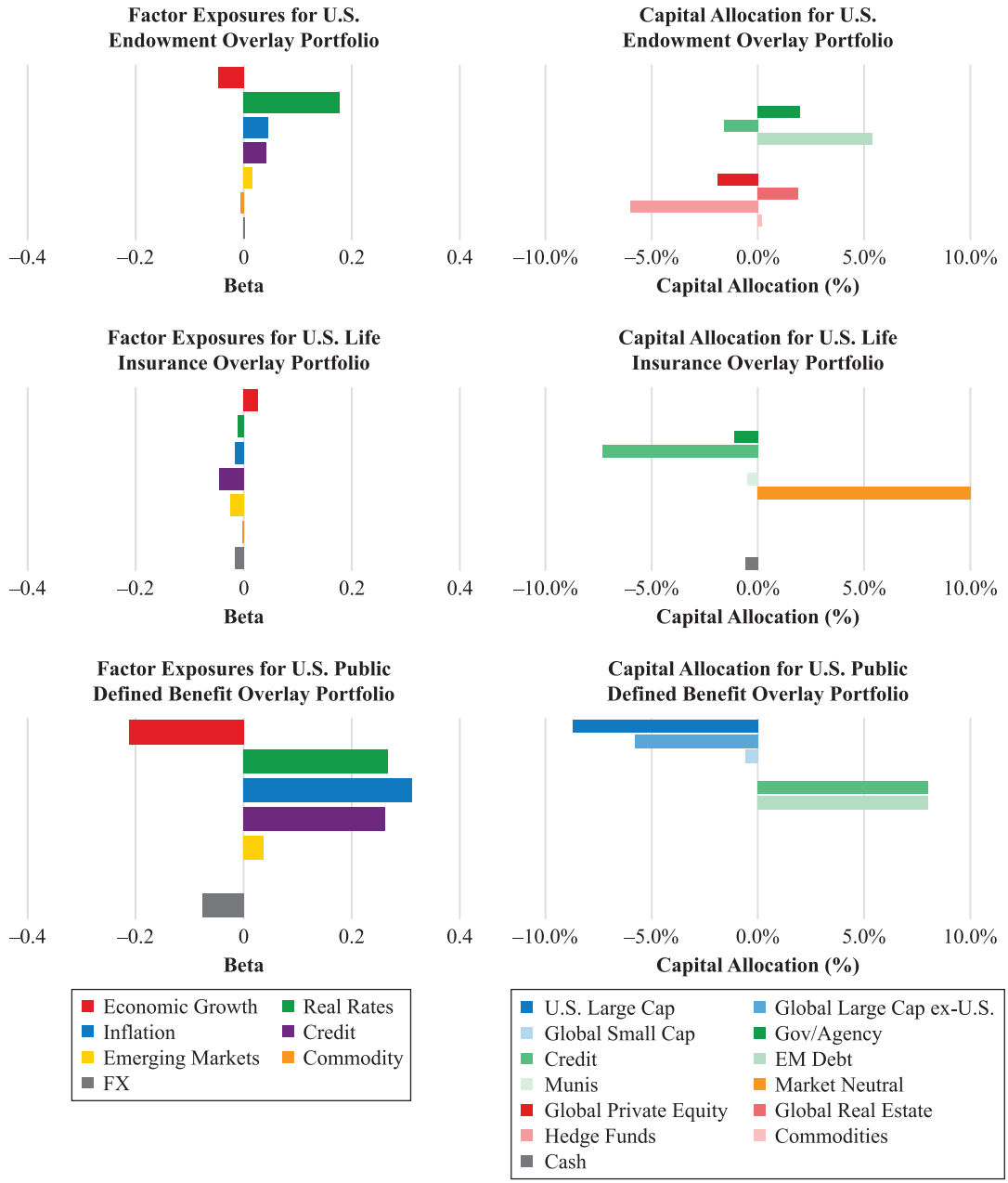
Source: Aladdin Factor Workbench, as of September 30, 2016.

In each case, we represent the overlay factor portfolios (see Exhibit 4), which result in improved risk–return ratios (see Exhibit 5), by a mix of traditional asset classes. It is important to note that we do not allocate to the asset classes directly—we hold asset classes as a means to obtaining the more balanced factor positions. We now discuss each of the modified asset class holdings and defer to the section that follows for examples of how to construct different combinations of asset classes to implement the new factor exposures.

U.S. endowment. We seek to reduce the risk of the U.S. endowment policy portfolio while maintaining long-term return expectations by rotating a growth-focused allocation into a factor-balanced blend.

Exhibit 4

Factor Completion Overlays for Representative Institutional Portfolios



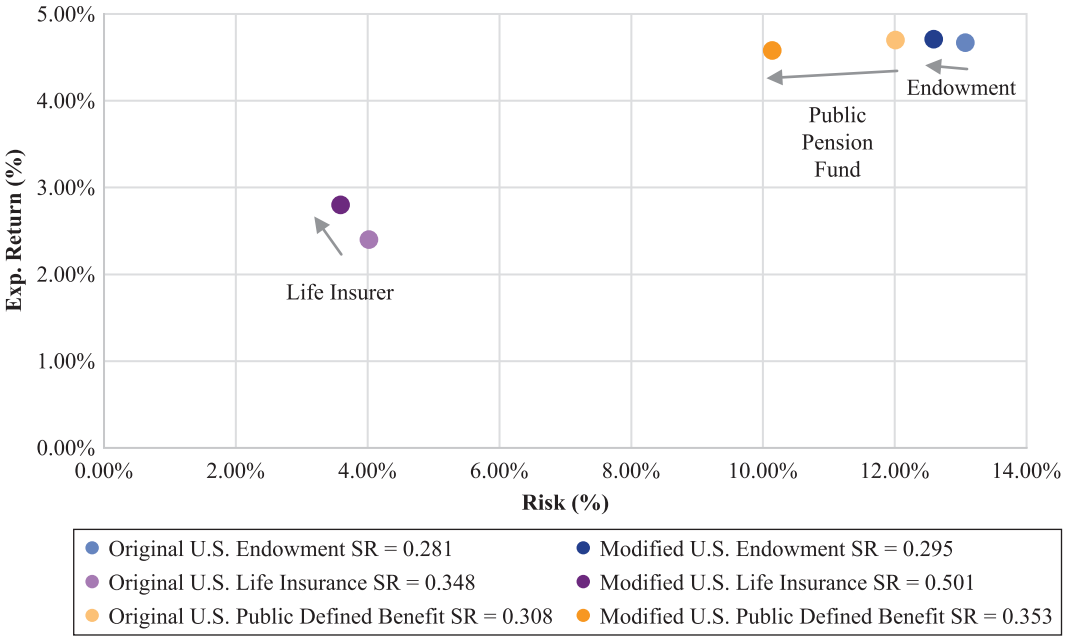
Note: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6.
Source: Aladdin Factor Workbench, as of September 30, 2016.

Exhibit 6 shows the capital allocation and macro factor risk contributions of the existing and proposed portfolios in Panels A and B, respectively.

The balanced factor portfolio reduces the risk contribution of the economic growth factor by 0.68% due to a shift from private equity and hedge funds into emerging market bonds, U.S. inflation-linked bonds, and real assets (i.e., private real estate). This transfer of assets represents a portfolio expense ratio reduction from 1.15% to 1.02%.[7] Such budget savings can be redirected

EXHIBIT 5

Hypothetical Risk–Return Characteristics of Representative Institutional Portfolios



Notes: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6. We use the long-term, annualized capital market assumptions produced by the BlackRock Investment Institute (www.blackrockblog.com/blackrock-capital-markets-assumptions) as of September 30, 2016. We use a U.S. cash expected return of 1.00% to calculate Sharpe ratios (SRs). We note that references to future returns are not promises or estimates of actual portfolio returns and are subject to uncertainty and error. Actual returns could be significantly higher or lower than forecasted. Capital market assumptions are provided for illustrative purposes only and should not be relied upon as recommendations to buy or sell securities. They are based on current market conditions and are subject to change without notice.

Source: Aladdin Factor Workbench, as of September 30, 2016.

toward alpha-seeking activities that pursue performance in markets with limited exposure to macro factors. There is a modest Sharpe ratio improvement from 0.28 to 0.30. Importantly, the balanced factor portfolio has maintained portfolio performance under the stress conditions shown in Panel C, with some improvements in conditions simulating the 2011 U.S. Downgrade and a global stock market drop due to larger real rates and inflation factor contributions.

Stress performance is assessed across the following three historical and three hypothetical scenarios: Credit 2007, Crash 2008, U.S. Downgrade 2011, Global Stock Market Drop, Rising U.S. Inflation, and Credible Fiscal Policy. Credit 2007 replicates the period July 1, 2007–July 1, 2008, where a credit and liquidity crisis caused by a severe slowdown in the U.S. housing market led to significant widening of credit spreads and increased implied volatility. Crash 2008 replicates September 12, 2008–November 3, 2008, and captures the equity market crash set off by Lehman Brothers' bankruptcy. U.S. Downgrade 2011 replicates July 21, 2011–September 20, 2011, when S&P downgraded U.S. credit and bond markets saw "flight to safety" gains. Global Stock Market Drop represents a 1% probability downward movement of MSCI World Market. Rising U.S. Inflation represents a 1% upward movement of CPI. Credible Fiscal Policy is a BlackRock Market-Driven Scenario that represents the hypothetical market impact of effective prospective fiscal policy in the United States.

U.S. life insurer. We seek to improve historical and hypothetical stress scenario performance for the representative U.S. life insurer by introducing a global, diversified, market-neutral investment strategy that respects the plan objectives of managing to liabilities and enhancing return while limiting equity market exposure. The majority of risk in insurers' portfolios stems from real rates, inflation, and credit; however,

EXHIBIT 6
Case: U.S. Endowment

Notes: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6. In Panel C, the stress performance for each scenario is displayed for the original portfolio (left) and modified portfolio (right); stress scenario descriptions are presented in the text. Stress-test performance is determined by the implied shock to each risk characteristic that the portfolio is exposed to. Relationships between risk characteristics and implied shocks are derived using historical correlations and BlackRock analysis.

Source: Aladdin Factor Workbench, as of September 30, 2016.

insurers have recently increased economic growth risk by investing in equities and alternatives in order to meet growing liabilities in a low interest rate environment. Allocating 10% of the policy portfolio to a diversified market-neutral strategy can reduce overall portfolio risk and lead to the possibility of enhanced expected return without introducing additional growth exposure.

EXHIBIT 7
Case: U.S. Life Insurance

Notes: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6. In Panel C, the stress performance for each scenario is displayed for the original portfolio (left) and modified portfolio (right); stress scenario descriptions are presented in the text. Stress-test performance is determined by the implied shock to each risk characteristic that the portfolio is exposed to. Relationships between risk characteristics and implied shocks are derived using historical correlations and BlackRock analysis.

Source: Aladdin Factor Workbench, as of September 30, 2016.

We show the modified capital allocation in Panel A and reduced risk contributions by all macro factors in Panel B of Exhibit 7, which, following Exhibit 5, shows a total portfolio risk reduction of 0.43% and return improvement of 0.40%. In Panel C, we observe that portfolio performance in the 2008 Crash scenario improves from −13.4% to −11.9%, performance during the 2007 Credit crisis improves from −8.8% to −8.2%,

and the modified portfolio remains comparably resistant to the hypothetical stock market drop, rising U.S. inflation, and fiscal policy scenarios.

U.S. public defined benefit plan. The typical U.S. public defined benefit plan portfolio has outsized economic growth exposure, which leaves it poorly diversified across return drivers and overexposed to equity market downside risk. By reducing the growth exposure and reallocating to other factors, the plan sponsor reduces sensitivity to equity market drawdowns and meets the target return with less risk.

Exhibit 8 compares the asset allocation, macro factor risk profiles, and performance of the plan under several stress scenarios when 20% of the equity allocation is replaced with a more balanced macro factor approach. Specifically, we allocate equally to emerging market bonds denominated in U.S. dollars and U.S. long credit. This improves the Sharpe ratio from 0.31 to 0.35. Moreover, portfolio performance under severe equity market stress improves due to the increased contribution of the credit and inflation factors. There is enhanced portfolio performance during scenarios calibrated to capture a Global Stock Market Drop (−6.7% to −5.6%) and the 2011 U.S. Downgrade (−10.1% to −7.4%). There is also not much of a performance give up during the market-driven scenario of Credible Fiscal Policy (2.5% to 1.7%).

FROM HERE TO THERE

While the previous section discussed the potential better risk–return outcomes with more balanced factor portfolios for the representative endowment, life insurer, and public defined benefit pension plan (in Exhibits 6 to 8, respectively), we illustrated those benefits with different combinations of asset classes that were able to obtain the new factor exposures. However, there could be other asset classes that are able to achieve the same factor exposures.

In this section, we take a more general approach to this final step in the strategic factor allocation framework: the determination of the optimal combination of asset classes, subject to investor preferences, that best implements the optimal factor exposures. This is an overspecified problem. With fewer factors than asset classes in total, there are an infinite number of asset class portfolios that can be constructed to match the desired factor mix. Thus, practical investor constraints are pivotal to identifying an optimal asset class portfolio.

This last step can be streamlined with an optimization procedure that maps factor exposures to asset allocations, whereby the asset portfolio is constructed bottom-up given the desired factor allocation and investor constraints on the investment universe. Such procedures (see, e.g., Blyth, Szigety, and Xia [2016]; Greenberg, Babu, and Ang [2016]) must minimize factor exposure deviations between the resulting asset portfolio and the target factor benchmark, respect a wide set of investor constraints, and be robust such that small changes in the desired factor mix produce comparably minor effects in the resulting asset allocation.

Mapping Factors to Assets

We leverage the robust optimization procedure formalized in Greenberg, Babu, and Ang [2016] to map the investor's optimal set of factor exposures onto an investable asset allocation that respects specified constraints. Our framework uses a robust optimization framework, in the sense that the objective function minimizes both tracking error and sum of squared exposure deviations between the target factor benchmark and the optimal asset portfolio. As in other applications, the robust optimization carries several advantages, including the solution not having undue sensitivity to small changes in inputs and minimizing the possibility of unintuitive corner solutions. The optimization problem further considers investor preferences including, but not limited to, expense, liquidity, holding size, leverage, turnover, risk, and return. The procedure handles both fully invested portfolios (optimal asset weights are constrained to sum to 100%) and overlay completion portfolios (optimal asset weights are constrained to sum to 0).[8]

To illustrate how we can use this robust optimization framework to obtain different sets of asset classes that implement the same factor exposures, we work with the U.S. public defined benefit plan. (Similar analyses can be produced for the other institutional portfolios). We construct two asset portfolios to implement our target factor exposures via optimization: mapping the desired factor exposures to only liquid assets and expanding the universe to include illiquid assets. The former was the case that we examined in Exhibit 8.

Exhibit 9 reports the results. Due to the constraints in the robust optimization, we do not completely match

EXHIBIT 8
Case: U.S. Public Defined Benefit Plan

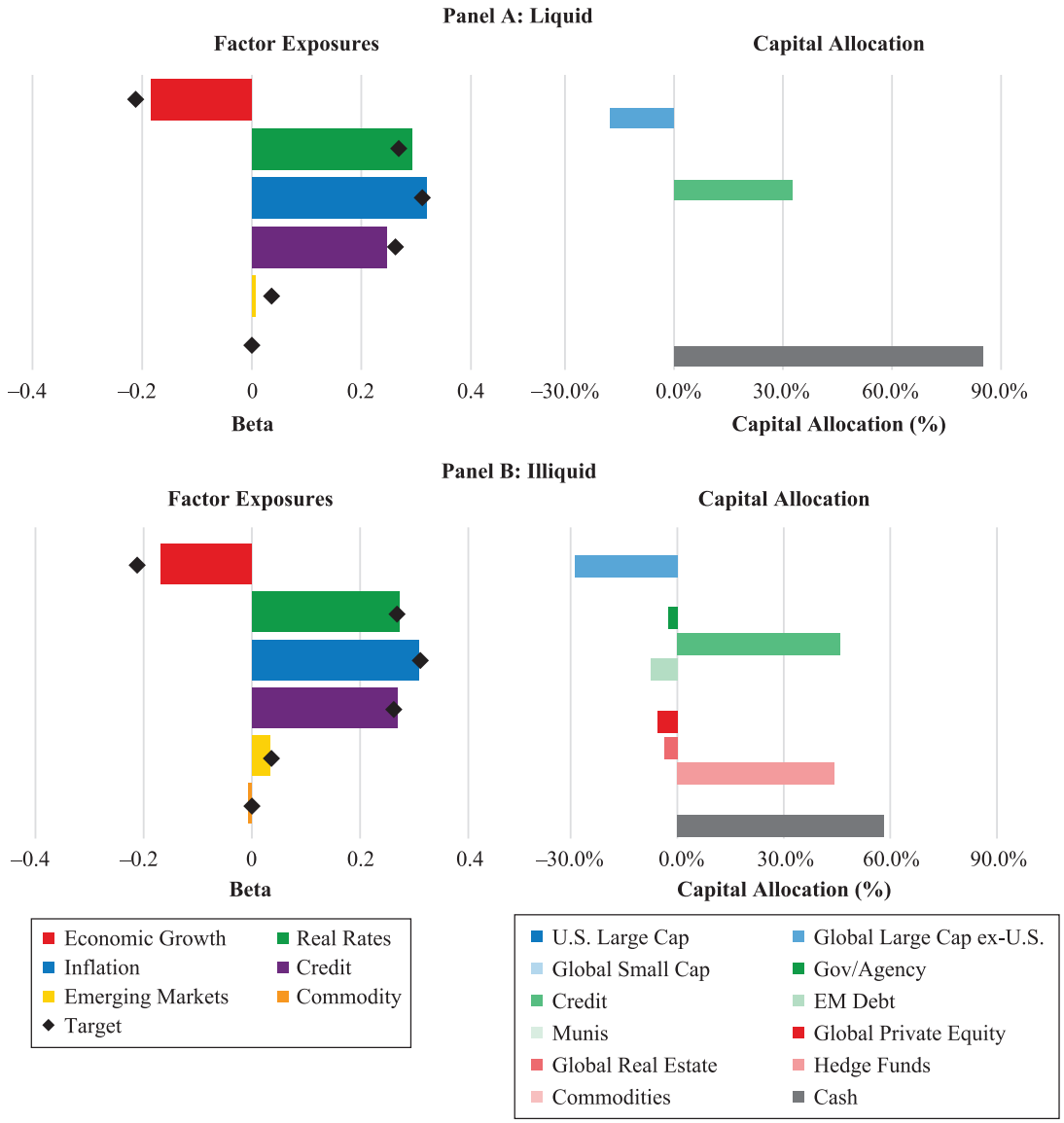
Notes: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6. In Panel C, the stress performance for each scenario is displayed for the original portfolio (left) and modified portfolio (right); stress scenario descriptions are presented in the text. Stress-test performance is determined by the implied shock to each risk characteristic that the portfolio is exposed to. Relationships between risk characteristics and implied shocks are derived using historical correlations and BlackRock analysis.

Source: Aladdin Factor Workbench, as of September 30, 2016.

the benchmark factor exposures, but the deviations from the target are small in both cases. In the illiquid asset case, the robust optimization chooses an asset portfolio consisting of Global Large Cap ex-U.S. (–28.8%), Gov/Agency (–2.5%), Credit (45.7%), EM Debt (–7.4%), Global Private Equity (–5.4%), Global Real Estate (–3.7%), Hedge Funds (4.4%), and Cash (58.0%). An (approximate) equivalent factor representation can

EXHIBIT 9

Liquid and Illiquid Completion Portfolio Implementations for U.S. Public Defined Benefit Plan



Notes: Hypothetical asset allocations for the representative institutions are constructed according to the methodology in Note 6. In Panel C, the stress performance for each scenario is displayed for the original portfolio (left) and modified portfolio (right); stress scenario descriptions are presented in the text. Stress-test performance is determined by the implied shock to each risk characteristic that the portfolio is exposed to. Relationships between risk characteristics and implied shocks are derived using historical correlations and BlackRock analysis.

Source: Aladdin Factor Workbench, as of September 30, 2016.

be obtained in the liquid asset classes through Global Large Cap ex-U.S. (–17.5%), Credit (32.5%), and Cash (85%). Note that the illiquid scenario requires more changes in the resulting asset allocation because those asset classes have more complex factor exposure profiles than their public market counterparts.

Using such a robust optimization-based approach to mapping factors to assets provides substantial flexibility to the investor. As market conditions change with time, there will be dynamic resulting asset allocations that reflect a given optimal factor set. Investors can set and maintain a strategic factor target that, subject to

a factors-to-assets mapping procedure, is responsive to changes in economic conditions and asset class features. The procedure can also assist with rebalancing, where asset class holding ranges can be combined with a modified factor benchmark in order to limit transaction costs and prevent undesired factor drift.

CONCLUSION

We present a strategic factor allocation framework across the total portfolio with the motivation of reframing asset allocation, portfolio analysis, and manager selection decisions along factor dimensions. There are three parts to the factor, not just asset, allocation framework: 1) measuring the factor exposures across all assets, with an emphasis on consistent treatment for liquid and illiquid markets; 2) determining optimal factor exposures based on criteria unique to each investor; and 3) determining the best mix of assets to implement a desired set of factor exposures subject to investor constraints. We emphasize the potential benefits of explicit diversification across factors—rather than just asset classes—and demonstrate modifications to typical institutional portfolios in factor space that can result in superior risk–return trade-offs.

Our framework is flexible across any set of factors and assets. Although we apply a robust constrained optimization procedure to map factor exposures onto asset allocations, we recognize that alternate optimization constructions or methodologies are also possible. Our treatment has also been technical, and there are additional management, governance, and real-world institutional constraints that need to be taken into account.

APPENDIX

MAPPING GRANULAR RISK EXPOSURES ONTO MACRO FACTORS

We translate a granular risk exposure view into a parsimonious macro factor view using constrained time-series regressions. Risk exposures are partitioned into blocks, and those blocks are separately regressed onto relevant macro factors. This effectively constrains the regressions to preserve economic intuition; for example, interest rate risk exposures are regressed only onto the real rates and inflation macro factors. In this section, we outline how to compute the exposures of a block of G risk exposures to F relevant macro factors.

We can express a time series of returns attributable to a block of exposures with N observations as follows:

$$\boldsymbol{y}_{block} = \alpha + R\boldsymbol{e} + \epsilon, \tag{A-1}$$

where

$\boldsymbol{y}_{block} = N \times 1$ *vector of block returns over time*
$\boldsymbol{e} = G \times 1$ *risk exposure loadings vector*
$R = N \times G$ *matrix of risk exposure returns*
$\boldsymbol{\alpha} = N \times 1$ *vector of alpha returns*
$\boldsymbol{\epsilon} = N \times 1$ *vector of errors*

On average, $\boldsymbol{\epsilon} = \mathbf{0}$, and we assume $\boldsymbol{\alpha} = \mathbf{0}$.

We compute the ordinary least squares (OLS) betas of $\boldsymbol{y}_{block}$ to the macro factors using:

$$\boldsymbol{\beta}_{block} = \left(\sum\right)^{-1} C_{xy}\, \boldsymbol{e}_{block}, \tag{A-2}$$

where

$\boldsymbol{\beta}_{block} = F \times 1$ *vector of exposures to the relevant macro factors*
$\Sigma = F \times F$ *invertible covariance matrix of relevant macro factors*
$C_{xy} = F \times G$ *cross-covariance matrix between macro factors and risk exposures*
$\boldsymbol{e}_{block} = G \times 1$ *vector of exposures to the risk exposures within the block*

Note that Equation A-2 holds because the residual under OLS is, by definition, uncorrelated to all regressors. We then sum blockwise betas to macro factors across all blocks to arrive at portfolio level macro exposures.

NON-EQUIVALENCE OF OPTIMAL ASSET CLASS AND FACTOR ALLOCATIONS

Define the K factor model over N returns:

$$\boldsymbol{R} = \boldsymbol{\mu} + B\boldsymbol{F} + \boldsymbol{\epsilon}, \tag{B-1}$$

where

$R = N \times 1$ *vector of asset class returns*
$B = N \times K$ *matrix of factor loadings*
$F = K \times 1$ *vector of factor returns for K factors*

Suppose investor has mean–variance utility, $U(E_p, \sigma_p^2)$, defined over portfolio expected returns, E_p, and return variance, σ_p^2. We maximize utility over portfolio weights, $\boldsymbol{w}$, with the portfolio adding-up constraint that $\boldsymbol{w}^T\mathbf{1} = 1$:

$$\max_{\boldsymbol{w}} U(E_p, \sigma_p^2) \tag{B-2}$$

The portfolio return is $\boldsymbol{R_p} = 1 + \boldsymbol{w^T}\boldsymbol{\mu} + \boldsymbol{w^T} B\boldsymbol{F} + \boldsymbol{w^T}\boldsymbol{\epsilon}$. For simplicity, we follow Ross [1978] with diversification in the limit, so we have $\boldsymbol{w^T}\boldsymbol{\epsilon} = 0$. (This is not strictly necessary and can be generalized.) We thus have

$$E_p = 1 + \boldsymbol{w^T}\boldsymbol{\mu},$$
$$\sigma_p^2 = \boldsymbol{w^T} B \sum\nolimits_F B^T \boldsymbol{w}. \qquad \text{(B-3)}$$

with Σ_F the $K \times K$ covariance matrix of the factors. Note that this does not involve asset-specific returns ("alphas"), giving rise to the third condition for equivalence between factors and asset views: that there are no asset-specific alphas.

We define the Lagrangian, with Lagrange multiplier λ, as follows:

$$U(E_p, \sigma_p^2) - \lambda(\boldsymbol{w^T}\boldsymbol{1} - 1) \qquad \text{(B-4)}$$

Taking partial derivatives and setting to zero, we have:

$$\frac{\partial U}{\partial E_p}\boldsymbol{\mu} + 2\frac{\partial U}{\partial \sigma_p^2} B \sum\nolimits_F B^T \boldsymbol{w} - \lambda = 0. \qquad \text{(B-5)}$$

We assume the existence of a zero-beta portfolio (with expected return equal to the risk-free rate) with weights $\boldsymbol{w_0}$ and mean $\boldsymbol{\mu_0}$. Premultiplying the first-order condition in Equation B-5 by $\boldsymbol{w_0}$, we obtain that $\lambda = U_1\boldsymbol{\mu}_0$. Subsequently substituting for the Lagrange multiplier and defining $\frac{1}{\gamma} = -\frac{U_1}{2U_2}$ as absolute risk aversion, we have

$$\boldsymbol{\mu} - \boldsymbol{\mu}_0 = \gamma B \sum\nolimits_F B^T \boldsymbol{w}. \qquad \text{(B-6)}$$

The optimal portfolio weights, $\boldsymbol{w}$, are implicitly defined by Equation B-6.

If we assume there are no leverage, shorting, or position constraints, we can define a factor with unconstrained linear transformations of the base assets. Define factor portfolio j with weights $\boldsymbol{q_j}$ such that $\mu_j = \boldsymbol{q_j^T}\boldsymbol{\mu}$. Collect these in a matrix $Q_F = [\boldsymbol{q_1}, \ldots, \boldsymbol{q_K}]$, so that we can define the factors F, where

$$F = Q_F^T \boldsymbol{R} \qquad \text{(B-7)}$$

Premultiplying Equation B-7 by Q_F gives

$$\boldsymbol{\mu}_F - \boldsymbol{\mu}_0 = \gamma Q_F^T B \sum\nolimits_F B^T \boldsymbol{w} \qquad \text{(B-8)}$$

Equation B-8 on factors is identical to Equation B-6 on assets. A trivial rotation is when the number of factors is equal to the number of assets, $K = N$.

We can derive a $K + 1$ mutual fund separation theorem (following Cass and Stiglitz [1970]) by solving for the holdings of the K factors directly, $\boldsymbol{q}$, where $\boldsymbol{q}$ is $K \times 1$. Rearranging Equation B-8, we have

$$\boldsymbol{q} = \gamma(B^T Q_F)^{-1} \sum\nolimits_F^{-1} (B^T B)^{-1} B^T (\boldsymbol{\mu_F} - \boldsymbol{\mu_0}). \qquad \text{(B-9)}$$

Note that Q_F and $\boldsymbol{q}$ will have long–short positions.

The two key assumptions in Equation B-9 are that the factors are formed entirely from the assets themselves and that we use unconstrained linear operations. If there are constraints, then we must project the factors to modify Equation B-6. Thus, the optimization problem on factors directly is not equivalent to performing the optimization problem in asset space. If there are any constraints (especially nonlinear constraints), then the linear algebra operations to produce Equation B-9 are not possible.

ENDNOTES

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[1]Economist Intelligence Unit [2016].

[2]The 13 global asset classes include inflation-linked debt, developed sovereign debt, IG (investment-grade) debt, EM (emerging market) sovereign debt, HY (high-yield) debt, developed equity, developed small-cap equity, EM equity, private equity, infrastructure, property, commodities ex-energy, and energy commodities. This time period across 13 assets was chosen as a practical trade-off between historical data availability and asset universe coverage.

[3]The macro factor correlations are nonzero empirically and should be nonzero theoretically. During times of economic expansion, for example, economic growth is high and inflation pressures increase. Importantly, the macro factor correlations are lower than the correlations of the asset class returns. Based on monthly returns from January 1997 to September 2015, the average cross-correlation of the macro factors is 0.23, whereas the average cross-correlation of the 13 global asset class returns is 0.44. The reduction in correlations is beneficial for constructing strategic portfolios in terms of factors rather than asset classes. The factor correlations do vary over time, which we can take into account by scenario analysis.

[4]Risk exposures that contribute less than 1% to the explanatory power of the regression are excluded from the mapping. The regression residuals are tracked and used in all covariance-based calculations. We source all asset return-time series from BlackRock's internal fundamental risk-factor model.

[5]Source: Aladdin Factor Workbench. Aladdin Factor Workbench is a proprietary BlackRock Solutions software intended to help users think about asset allocation through a macro factor lens; it takes a user-constructed portfolio and performs a risk and return analysis that includes a calculation of the decomposition of risk across both asset classes and macro factors.

[6]Sample allocations are hypothetical portfolios illustrative of the specified types of institutions. The U.S. Endowment composite portfolio is based on aggregate analysis of asset allocations in the 2014 NACUBO study. The U.S. Life Insurance industry portfolio is constructed by BlackRock using CUSIP-level statutory NAIC filings of over 150 life insurance companies in 2015 covering USD 3.4 trillion in asset classes. The U.S. Public Defined Benefit plan peer composite is constructed using asset allocations and data sourced from the Pensions & Investments Research Center, specifically the P&I 2014 Top 1000 Retirement Funds.

[7]Expense ratios for asset classes are representations of the typical cost of publicly tradable instruments and expense expectations of private market offerings. Expenses for asset classes are estimated as follows for the purpose of this analysis: U.S. large cap, 0.05%; global large cap ex-U.S., 0.14%; government/agency, 0.15%; credit, 0.20%; EM debt, 0.50%; global private equity, 2.50%; global real estate, 2.50%; hedge funds, 2.50%; commodities, 1.00%; and cash, 0.00%.

[8]One advantage of factor-based overlay strategies, such as those discussed in the institutional case studies discussed here, is they allow for plan-level offsets of undesired factor exposures without disrupting an existing policy or introducing additional transaction costs.

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